A FUZZY COMPARISON METHOD FOR PARTICULAR FUZZY NUMBERS

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ABSTRACT. In a previous work, we introduced particular fuzzy numbers and discussed some of their properties. In this paper we use the comparison method introduced by Dorohonceanu and Marin[5] to compare between these fuzzy numbers.

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1. INTRODUCTION

Fuzzy Set Theory considers ambiguity and imprecision. In Set Theory, representations are forced to comply with precise models; it avoids and rejects imprecision as a factor. Imprecision plays an important role in information presentation in real processes. Fuzzy Set Theory allows the formalization of approximate reasoning and preserves the original information contents of imprecision [10]. Fuzzy Set Theory and its Applications has been an interesting area of research in the last few decades for many authors and many articles were published since introducing the

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concepts of Fuzzy sets and Probability Measure of Fuzzy Events by L. A. Zadeh [8] and [9], respectively. In particular, Fuzzy Number Comparison Methods have been researched [2]-[5].

In many real applications, there are some imprecise data represented by fuzzy numbers. Recently, fuzzy numbers are largely applied in data analysis, artificial intelligence and decision making. In [7], we introduced particular fuzzy numbers, namely Trapezoidal-Parabolic Fuzzy Numbers, Trapezoidal-Parabolic shaped Fuzzy Numbers, Parabolic Fuzzy Numbers and Parabolic shaped Fuzzy Numbers. In all cases the fuzzy numbers are determined by particular points; we also mentioned some of their properties. Dadgostar [3] proposed a fuzzy number comparison method called Partial Comparison Method. It is based on the fuzzy number division in comparison, the shapes of the convex numbers do not require special computations during comparison because it relies on the representation of fuzzy numbers as ordered sets of confidence intervals. Dorohonceanu and Marin [5] presented a fuzzy number comparison method based on the fuzzy number representations in fuzzy arithmetic described in [3]. They also described a variant based on the fuzzy number division in comparison used in partial comparison method. Triangular and trapezoidal shapes of membership functions were used to describe the method.

In this paper, we apply the method presented in [5] to the fuzzy numbers introduced in [7].

2. Fuzzy Sets and Numbers

Definition 2.1. A fuzzy subset A of some set Ω is defined by its membership function written $A(x)$which produces values in $[0, 1]$ for all x in Ω. That is $A(x)$ is a function mapping Ω into $[0, 1]$. We place a bar over a letter to denote a fuzzy set, that is $\overline{\overline{A}}$. The term crisp means not fuzzy. A crisp set is a regular set.

Definition 2.2. Let $\Omega = R$. An $\alpha-$cut of $\overline{\overline{A}}$, written $\overline{\overline{A}}[\alpha]$, is defined as $\{x : \overline{\overline{A}}(x) \geq \alpha\}$, for $0 < \alpha \leq 1$. $\overline{\overline{A}}[0]$, the support of $\overline{\overline{A}}$ is defined as the closure of the union of all the $\overline{\overline{A}}[\alpha]$, for $0 < \alpha \leq 1$.

Definition 2.3. A confidence interval is an interval of real numbers that provides a representation for an imprecise numerical value by means of its sharpest enclosing range.

Definition 2.4. A presumption level is an estimated truth-value about some knowledge. Presumption levels belong to the [0,1] interval; the maximum of estimated truth-value is at level 1 and the minimum is at level 0.

Definition 2.5. A fuzzy number $\overline{\overline{N}}$ is a fuzzy subset of the real numbers satisfying:

(1) $\exists x \in R : \overline{\overline{N}}(x) = 1.$

$\overline{\overline{N}}[\alpha]$ is a closed and bounded interval for $0 \leq \alpha \leq 1$.

A special type of fuzzy numbers $\overline{\overline{M}}$ is called a triangular fuzzy number. $\overline{\overline{M}}$ is defined by three numbers $a_1 < a_2 < a_3$ where (1) $\overline{\overline{M}}(x) = 1$at $x = a_2$. (2) The graph of $\overline{\overline{M}}(x)$ on $[a_1, a_2]$ is a straight line from $(a_1, 0)$ to $(a_2, 1)$ and also on $[a_2, a_3]$ the graph is a straight line from $(a_2, 1)$ to $(a_3, 0)$. (3) $\overline{\overline{M}}(x)$ for $x \leq a_1$or $x \geq a_3$. We write $\overline{\overline{M}} = (a_1/a_2/a_3)$ for triangular fuzzy number $\overline{\overline{M}}$. If at least one of the graphs described above is not a straight line (curve), then $\overline{\overline{M}}$ is called triangular shaped fuzzy number and we write

$$\overline{\overline{M}} \approx (a_1/a_2/a_3).$$

Another special type of fuzzy numbers $\overline{\overline{M}}$ is called a trapezoidal fuzzy number. $\overline{\overline{M}}$ is defined by four numbers $a_1 < a_2 < a_3 < a_4$ where (1) $\overline{\overline{M}}(x) = 1$ on $[a_2, a_3]$. (2) The graph of $\overline{\overline{M}}(x)$ on $[a_1, a_2]$ is a straight line from $(a_1, 0)$ to $(a_2, 1)$ and also on $[a_3, a_4]$ the graph is a straight line from $(a_3, 1)$ to $(a_4, 0)$. (3) $\overline{\overline{M}}(x)$ for $x \leq a_1$or $x \geq a_4$. We write $\overline{\overline{M}} = (a_1, a_2, a_3, a_4)$ for trapezoidal fuzzy number $\overline{\overline{M}}$. If at least one of the graphs described above is not a straight line (curve), then M is called trapezoidal shaped fuzzy number and we write $\overline{\overline{M}} \approx (a_1, a_2, a_3, a_4)$.

In [7], we introduced other types of fuzzy numbers. We considered a fuzzy number that is determined by five real numbers a_1, a_2, a_3, a_4 and c such that $a_1 < a_2 < a_3 < a_4$ and $0 < c < 1$, denoted by $\overline{\overline{N}}_c = (a_1/a_2/a_3/a_4)$ or $(a_1, a_2, a_3, a_4)_c$ whose membership function is given by

$$\overline{\overline{N}}_c(x) = \begin{cases} 0, & x \leq a_1 \\ \frac{c}{a_2-a_1}(x-a_1), & a_1 \leq x \leq a_2 \\ 1 - \frac{1-c}{(a_2-a_3)^2}(2x-a_2-a_3)^2, & a_2 \leq x \leq a_3 \\ \frac{-c}{a_4-a_3}(x-a_4), & a_3 \leq x \leq a_4 \\ 0, & x \geq a_4 \end{cases}$$

(1) $\overline{\overline{N}}_c(x)$is a line from $(a_1, 0)$ to (a_2, c).

(2) $\overline{\overline{N}}_c(x)$is a parabola from (a_2, c) to (a_3, c) whose vertex is $\left(\frac{a_2+a_3}{2}, 1\right)$ and focus is

$$\left(\frac{a_2 + a_3}{2}, 1 - \frac{(a_3 - a_2)^2}{16(1-c)}\right).$$

(3) $\overline{\overline{N}}_c(x)$is a line from (a_3, c) to $(a_4, 0)$.

We called such a fuzzy number a Trapezoidal-Parabolic Fuzzy Number and if it is determined by five numbers and it is not of this form, we called it a Trapezoidal-Parabolic Shaped Fuzzy Number.

If we put $c = 0$ in the membership function of the trapezoidal-parabolic fuzzy number, then we get another fuzzy number. We called it parabolic fuzzy number, and is defined by two real numbers a_1 and a_2 with $a_1 < a_2$, denoted by $\overline{\overline{N}} = (a_1/a_2)$ or $\overline{\overline{N}} = (a_1, a_2)$. The membership function $\overline{\overline{N}}(x)$ is given by

$$\overline{\overline{N}}(x) = \begin{cases} 0, & x \leq a_1 \\ 1 - \frac{1}{(a_1-a_2)^2}(2x - a_1 - a_2)^2, & a_1 \leq x \leq a_2 \\ 0, & x \geq a_2 \end{cases}$$

(1) $\overline{\overline{N}}(x)$ is parabolic in $[a_1, a_2]$ whose vertex is $\left(\frac{a_1+a_2}{2}, 1\right)$ and focus

$$\left(\frac{a_1 + a_2}{2}, 1 - \frac{(a_1 - a_2)^2}{16}\right).$$

(2) If $\overline{\overline{N}}(x)$ is not parabolic in $[a_1, a_2]$ then it is called a parabolic shaped fuzzy number and is denoted by $\overline{\overline{N}} \approx (a_1/a_2)$ or $\overline{\overline{N}} \approx (a_1, a_2)$.

(3)

$$\overline{\overline{N}}[\alpha] = \left[\frac{-(a_2 - a_1)}{2}\sqrt{1-\alpha} + \frac{a_2 + a_1}{2}, \frac{(a_2 - a_1)}{2}\sqrt{1-\alpha} + \frac{a_2 + a_1}{2}\right]$$

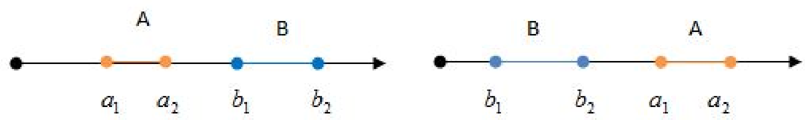

FIGURE 1. *A is greater than B with $\alpha_{A>B} = 0$ and $\alpha_{A>B} = c$, respectively.*

for $0 \leq \alpha \leq 1$.

(4) $\overline{\overline{N}} \approx \left(a_1 / \frac{a_1+a_2}{2} / a_2\right)$ is a triangular shaped fuzzy number.

A fuzzy number is represented as an ordered set of confidence intervals, each of them provides the related numerical value at a given presumption level $\alpha_1 \in [0,1]$. These confidence intervals should comply with the relation if $\alpha_1 > \alpha_2$ then $A_{\alpha_1} \subset A_{\alpha_2}$, where $\alpha_1, \alpha_2 \in [0,1]$ and $A_{\alpha_1},\ A_{\alpha_2}$ are the confidence intervals at presumption levels α_1 and α_2 respectively. More details, properties and operations, can be found in [1], [6] and [11].

3. Confidence Interval Comparison

As mentioned in sec 2, the maximum of estimated truth-value is at level 1 and the minimum is at level 0. Here we apply the method in [5] assuming that the maximum (the minimum) is c where $0 < c \leq 1,\ 0 \leq c < 1$ respectively.

Let us consider two confidence intervals $A = (a_1, a_2)$ and $B = (b_1, b_2)$. If A and B do not overlap, we can say $A > B$ with $\alpha_{A>B} = c$ or $\alpha_{A>B} = 0$ (Figure 1). If A and B overlap, then $\alpha_{A>B} \in (0, c)$. In the latter case, we translate A on the right side of B to a point from which the two intervals do not overlap (Figure2), we can say the new fuzzy number $A_R > B$ with $\alpha = c$. And if we translate A on the left side of B to a point from which the two intervals do not overlap, we can say the new fuzzy number $A_L > B$ with $\alpha = 0$.

Triangular similarity (Figure 3) implies

$$\frac{\alpha_{A>B}}{c} = \frac{a_1 - (b_1 - (a_2 - a_1))}{b_2 - b_1 + a_2 - a_1} = \frac{a_2 - b_1}{b_2 - b_1 + a_2 - a_1}.$$

Thus

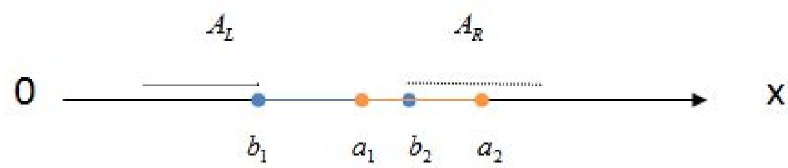

FIGURE 2. *Computing $\alpha_{A>B} \in (0, c)$ for interval A.*

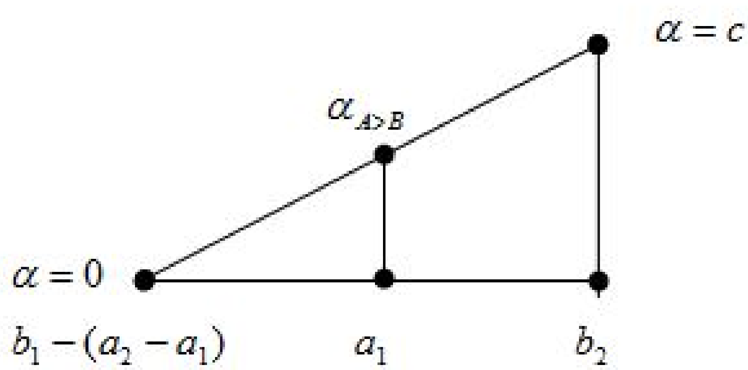

FIGURE 3.

$$\alpha_{A \geq B} = \frac{c(a_2 - b_1)}{b_2 - b_1 + a_2 - a_1}$$

Similarly, if $A = (a_1, a_2)$ and $B = (b_1, b_2)$, when $\alpha_{A>B} \in (c, 1)$, then

$$\alpha_{A>B} = \frac{(1-c)(a_2 - b_1)}{a_2 - a_1 + b_2 - b_1}.$$

Now

$$\alpha_{A>B} + \alpha_{B>A} = c \tag{1}$$

and

$$\alpha_{A>B} + \alpha_{B>A} = 1 - c \tag{2}$$

Adding (1) and (2) we get

$$\alpha_{A>B} + \alpha_{A>B} + \alpha_{B>A} + \alpha_{B>A} = 1 \tag{3}$$

Hence, equation (3) stands for the comparison done in two steps.

4. Fuzzy Number Comparison

The variant of the B2 method B2x [5] is based on the fuzzy number division in comparison used in PCM method [3]. This method has the idea that (when comparing triangular fuzzy numbers) each part of constant monotony of fuzzy number A is separately compared with fuzzy number B. We divide A into two fuzzy intervals : A_1 (increasing line),A_2 (decreasing line). We compare A_1 with fuzzy number B by finding the truth value of is greater than B ($\alpha_{A_1>B}$) and with fuzzy number B by finding the truth value of is greater than B ($\alpha_{A_2>B}$) . At the end we find the true value of A is greater than B ($\alpha_{A>B} = \frac{\alpha_{A_1>B}+\alpha_{A_2>B}}{2}$) and the true value of B is greater than A ($\alpha_{B>A} = 1 - \alpha_{A>B}) Now, if \alpha_A > B > \alpha_B > A$ then $A > B$. The B2x method is used to compare between trapezoidal fuzzy numbers and triangular fuzzy number and trapezoidal fuzzy number. Using this method, we will compare between trapezoidal- parabolic fuzzy numbers $\overline{A_c} = (a_1, a_2, a_3, a_4)$ and $\overline{B_c} = (b_1, b_2, b_3, b_4)$, where the membership functions of $\overline{A_c} and \overline{B_c}$ are mentioned in sec 2.

The first step is approximating the trapezoidal- parabolic fuzzy numbers to piece-wise linear functions. We Approximate $\overline{A_c}$ to $\overline{A}$ and $\overline{B_c}$ to $\overline{B}$.

Where

$$\overline{A}(x) = \begin{cases} 0, & x \leq a_1 \\ \frac{c}{a_2-a_1}(x-a_1), & a_1 \leq x \leq a_2 \\ \frac{2(1-c)}{a_3-a_2}(x-a_2)+c, & a_2 \leq x \leq \frac{a_3+a_2}{2} \\ \frac{2(c-1)}{a_3-a_2}(x-a_3)+c, & \frac{a_3+a_2}{2} \leq x \leq a_3 \\ \frac{-c}{a_4-a_3}(x-a_4), & a_3 \leq x \leq a_4 \\ 0, & x \geq a_4 \end{cases}$$

Similarly, $\overline{B_c}$ to $\overline{B}$, replacing a_i by b_i (Figure 4).

Now, if $\overline{A}$ and $\overline{B}$ do not overlap then $\alpha_{\overline{A}>\overline{B}} = 0$ or $\alpha_{\overline{A}>\overline{B}} = 1$, otherwise $\alpha_{\overline{A}} > \overline{B} \in (0, 1)$.

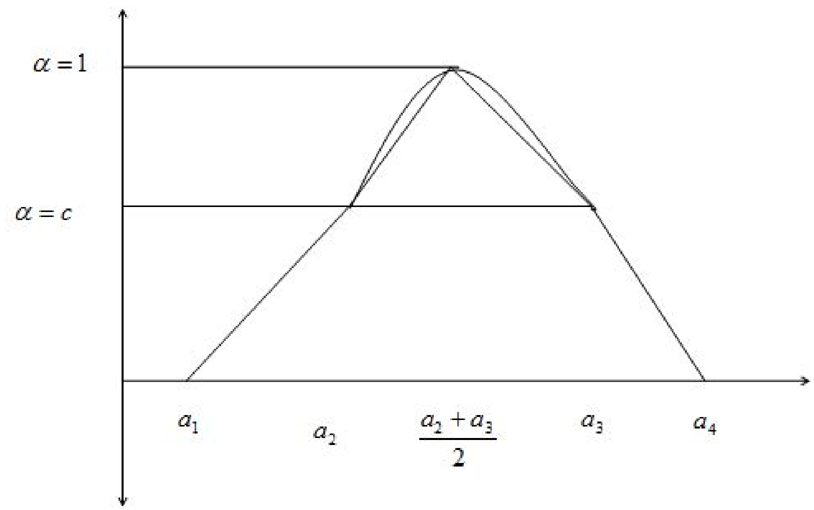

FIGURE 4.

Next, divide $\overline{A}$ into two intervals, $\overline{A_{1'}}$, $\overline{A_{2'}}$and $\overline{B}$ into two $\overline{B_{1'}}$, $\overline{B_{2'}}$(Figure 5). Now, the comparison will be in two parts, $\overline{A_{1'}}$with $\overline{B_{1'}}$and $\overline{A_{2'}}$with $\overline{B_{2'}}$. We Divide $\overline{A_{1'}}$into three intervals $\overline{A_{1''}}$, $\overline{A_{2''}}$, $\overline{A_{3''}}$and $\overline{B_{1'}}$into three intervals $\overline{B_{1''}}$, $\overline{B_{2''}}$, $\overline{B_{3''}}$(Figure 6).

Part1:

Comparing $\overline{A_{1''}}$with $\overline{B_{1'}}$:

a): $\alpha_{\overline{A_{1''}}>\overline{B_{1''}}} = 0.$

b): $\alpha_{\overline{A_{1''}}>\overline{B_{2''}}} = 0.$

c): $\alpha_{\overline{A_{1''}}>\overline{B_{3''}}} = 0.$

Hence

$$\alpha_{\overline{A_{1''}}>\overline{B_{1'}}} = \frac{\alpha_{\overline{A_{1''}}>\overline{B_{1''}}} + \alpha_{\overline{A_{1''}}>\overline{B_{2''}}} + \alpha_{\overline{A_{1''}}>\overline{B_{3''}}}}{3} = \frac{0+0+0}{3} = 0.$$

Comparing $\overline{A_{2''}}$ with $\overline{B_{1'}}$:

a):

$$\alpha_{\overline{A_{2''}}>\overline{B_{1''}}} = \frac{c(a_3-b_1)}{b_2-b_1+a_3-a_2}$$

b):

$$\alpha_{\overline{A_{2''}}>\overline{B_{2''}}} = \frac{c(a_3-b_2)}{b_3-b_2+a_3-a_2}$$

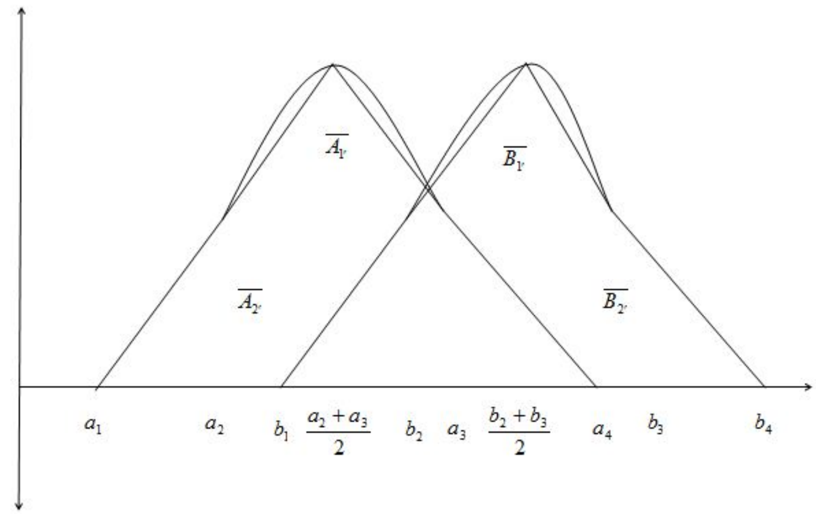

FIGURE 5.

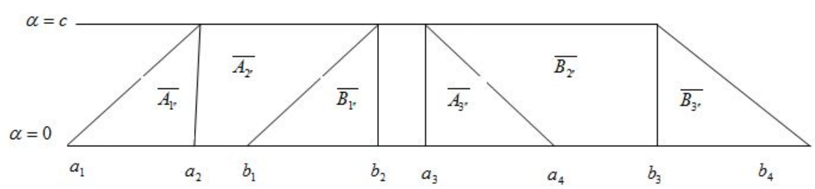

FIGURE 6.

c): $\alpha_{\overline{A_{2''}}>\overline{B_{3''}}} = 0$

Hence

$$\alpha_{\overline{A_{2''}}>\overline{B_{1'}}} = \frac{\alpha_{\overline{A_{2''}}>\overline{B_{1''}}} + \alpha_{\overline{A_{2''}}>\overline{B_{2''}}} + \alpha_{\overline{A_{2''}}>\overline{B_{3''}}}}{3}.$$

Comparing $\overline{A_{3''}}$ with $\overline{B_{1'}}$

a): $\alpha_{\overline{A_{3''}}>\overline{B_{1''}}} = c$

b):

$$\alpha_{\overline{A_{3''}}>\overline{B_{2''}}} = \frac{c(a_4 - b_2)}{b_3 - b_2 + a_4 - a_3}$$

c): $\alpha_{\overline{A_{3''}}>\overline{B_{3''}}} = 0$

Hence

$$\alpha_{\overline{A_{3''}}>\overline{B_{1'}}} = \frac{\alpha_{\overline{A_{3''}}>\overline{B_{1''}}} + \alpha_{\overline{A_{3''}}>\overline{B_{2''}}} + \alpha_{\overline{A_{3''}}>\overline{B_{3''}}}}{3}.$$

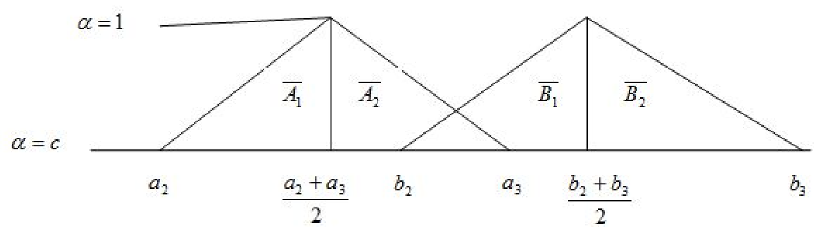

FIGURE 7.

Finally, the membership of $\overline{A_{1'}} > \overline{B_{1'}}$

$$\alpha_{\overline{A_{1'}}>\overline{B_{1'}}} = \frac{\alpha_{\overline{A_{1''}}>\overline{B_{1'}}} + \alpha_{\overline{A_{2''}}>\overline{B_{1'}}} + \alpha_{\overline{A_{3''}}>\overline{B_{1'}}}}{3}$$

Part 2:

Compare $\overline{A_{2'}}$with $\overline{B_{2'}}$. Divide $\overline{A_{2'}}$ into $\overline{A_1}$ and $\overline{A_2}$ and $\overline{B_{2'}}$ into $\overline{B_1}$ and $\overline{B_2}$ (Figure 7).

Compare $\overline{A_1}$ and $\overline{B_{2'}}$

a): $\alpha_{\overline{A_1}>\overline{B_1}} = 0.$

b): $\alpha_{\overline{A_1}>\overline{B_2}} = 0$

Hence

$$\alpha_{\overline{A_1}>\overline{B_{2'}}} = \frac{0+0}{2} = 0.$$

Compare $\overline{A_2}$ with $\overline{B_{2'}}$

a):

$$\alpha_{\overline{A_2}>\overline{B_1}} = \frac{(1-c)(a_3-b_2)}{a_3 - (\frac{a_2+a_3}{2}) + (\frac{b_2+b_3}{2}) - b_2}$$

b): $\alpha_{\overline{A_2}>\overline{B_2}} = 0.$

Hence

$$\alpha_{\overline{A_2}>\overline{B_{2'}}} = \frac{\alpha_{\overline{A_2}>\overline{B_1}} + \alpha_{\overline{A_2}>\overline{B_2}}}{2}.$$

Finally, the membership of $\overline{A_{2'}} > \overline{B_{2'}}$

$$\alpha_{\overline{A_{2'}}>\overline{B_{2'}}} = \frac{\alpha_{\overline{A_1}>\overline{B_{2'}}} + \alpha_{\overline{A_2}>\overline{B_{2'}}}}{2}$$

$$\alpha_{\overline{A}>\overline{B}} = \alpha_{\overline{A_{1'}}>\overline{B_{1'}}} + \alpha_{\overline{A_{2'}}>\overline{B_{2'}}}$$

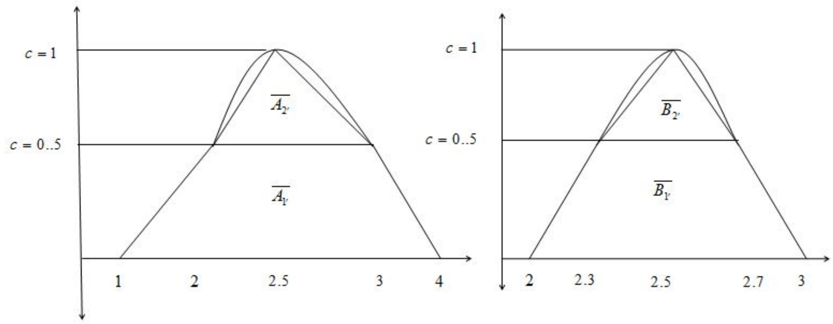

FIGURE 8.

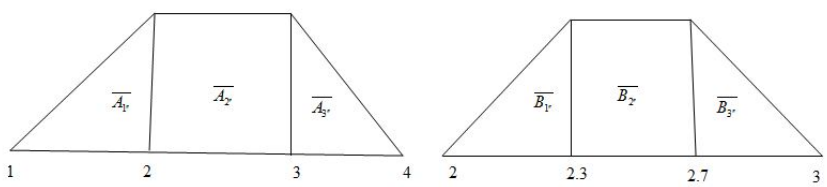

FIGURE 9.

$$\alpha_{\overline{A}>\overline{B}} = 1 - \alpha_{\overline{B}>\overline{A}}$$

If $\alpha_{\overline{A}>\overline{B}} > \alpha_{\overline{B}>\overline{A}}$ then $\overline{A} > \overline{B}$.

5. Numerical Example

Consider the fuzzy numbers in Figures 8-10

Comparing $A_{1''}$ with $B_{1'}$, we get

$$\alpha_{A_{1''}>B_{1''}} = 0,\ \alpha_{A_{1''}>B_{2''}} = 0 \text{ and } \alpha_{A_{1''}>B_{3''}} = 0 \Rightarrow \alpha_{A_{1''}>B_{1'}} = \frac{0+0+0}{3} = 0.$$

Comparing $A_{2''}$ with $B_{1'}$, we get

$$\alpha_{A_{2''}>B_{1''}} = \frac{(3-2)\,(0.5)}{3-2+2.3-2} = \frac{5}{13},$$

$$\alpha_{A_{2''}>B_{2''}} = \frac{(3-2.3)\,(0.5)}{3-2+2.7-2.3} = \frac{35}{150}$$

and

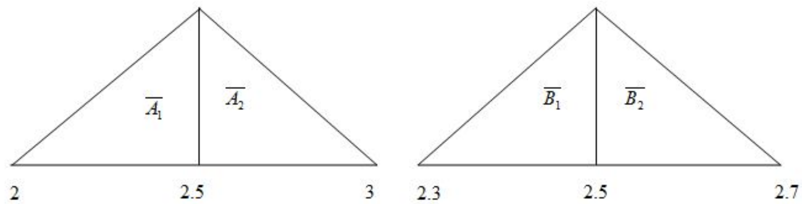

Figure 10.

$$\alpha_{A_{2''}>B_{3''}} = \frac{(3-2.7)(0.5)}{3-2+3-2.7} = \frac{15}{130}$$

which implies

$$\alpha_{A_{2''}>B_{1'}} = \frac{\frac{5}{13}+\frac{35}{150}+\frac{15}{130}}{3} = 0.2444$$

Comparing $A_{3''}$ with $B_{1'}$, we get $\alpha_{A_{3''}>B_{1''}} = c = 0.5$, $\alpha_{A_{3''}>B_{2''}} = c = 0.5$ and $\alpha_{A_{3''}>B_{3''}} = c = 0.5, \Rightarrow \alpha_{A_{3''}>B_{1'}} = 0.5$

Thus

$$\alpha_{A_{1'}>B_{1'}} = \frac{\alpha_{A_{1''}>B_{1'}}+\alpha_{A_{2''}>B_{1'}}+\alpha_{A_{3''}>B_{1'}}}{3} = \frac{0+0.2444+0.5}{3} = 0.24815$$

Comparing $A_{2'}$ with $B_{2'}$, we divide $A_{2'}$ into A_1 and A_2 and $B_{2'}$ into B_1 and B_2. Now we compare A_1 with $B_{2'}$,

$$\alpha_{A_1>B_1} = \frac{(1-0.5)(2.5-2.3)}{2.5-2-2.5-2.3} = \frac{1}{7} \text{ and } \alpha_{A_1>B_2} = 0$$

leading to

$$\alpha_{A_1>B_{2'}} = \frac{\frac{1}{7}+0}{2} = \frac{1}{14}.$$

Comparing A_2 with $B_{2'}$,

$$\alpha_{A_2>B_1} = (1-0.5) = 0.5$$

and

$$\alpha_{A_2>B_2} = \frac{(1-0.5)(3-2.5)}{3-2.5+2.7-2.5} = \frac{25}{70}$$

Leading to

$$\alpha_{A_2 > B_{2'}} = \frac{0.5 + \frac{25}{70}}{2} = 0.428571428.$$

Thus

$$\alpha_{A_{2'} > B_{2'}} = \frac{\alpha_{A_1 > B_{2'}} + \alpha_{A_2 > B_{2'}}}{2} = \frac{0.428571428 + \frac{1}{14}}{2} = 0.25$$

Finally,

$$\alpha_{A' > B'} = \alpha_{A_{1'} > B_{1'}} + \alpha_{A_{2'} > B_{2'}} = 0.4944$$

and

$$\alpha_{B' > A'} = 1 - \alpha_{A' > B'} = 0.5056.$$

6. Conclusion

We were able to apply the method of [5] to the particular fuzzy numbers in [7]. Since we used approximation in this work, there is a numerical error in our estimated truth-value. We will try to find a formula for this error in further work. We will also try to use these numbers in many applications where triangular and trapezoidal fuzzy numbers are used, the first step of that is comparing them which we did in this work.

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