PROCESS CONTROL USING ASSUMED FUZZY TEST AND FUZZY ACCEPTANCE REGION

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Abstract. There are many situations for statistical process in which we have both random and vague information. When uncertainty is due to fuzziness of information, fuzzy statistical control charts play an important role in the monitoring process, because they simultaneously deal with both kinds of uncertainty. Dealing with fuzzy characteristics using classical methods may cause the loss of information and influence in process deciding making. In this paper, we proposed a decision-making process based on fuzzy rejection regions and fuzzy statistical tests for crisp observation. With both methods, we define the degree of dependence to acceptance region for decision in the fuzzy regions and process fuzzy. A numeric example illustrates the performance of the method and interprets the results.

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1. Introduction

Hypotheses testing methods are extensively used in various statistical quality control. These methods may be used to infer whether there is conformity between process parameters and their specified values and/or whether they can help to modify the process to achieve a desired value. Faraz et al. in their article presented an application of fuzzy random variables in control charts and the structure fuzzy of Shewhart control charts [4,5,6]. In this regard, Zarandi et al. [21], suggested a hybrid approach based on fuzzy sampling rules. Kaya and Kahraman[11], Glbay and Kahraman [7,8], used of fuzzy set theory for the construction of fuzzy control

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charts. Wang and Raz [17,18], proposed two fuzzy approaches, called the membership function approach and the fuzzy-probabilistic approach for monitoring the process average. Using defuzzification methods, fuzzy sample data are first converted into real-valued sample data, then the center line and control limits are established as in e traditional Shewhart control charts.

Let $X_1, X_2, \ldots, X_n$ be a random sample observed from a normal distribution $N(\mu, \sigma^2)$, where σ is known. The objective is to test the hypotheses $H_\circ : \mu = \mu_\circ$, against$H_1 : \mu \neq \mu_\circ$. If the null hypotheses is true, the statistic $Z = \frac{\overline{X} - \mu_\circ}{\frac{\sigma}{\sqrt{n}}}$, has a standard normal distribution $N(0, 1)$.

Therefore, the rejection region with significant level α is $R = \{Z \mid |Z| \geq Z_\alpha\}$, where $P(|Z| \geq Z_\alpha) = \alpha$[1].

2. Fuzzy Hypotheses

Sometimes, the nature of such hypotheses is such that it cannot be formulated in a precise terms. In this case the theory of fuzzy sets can be used in hypotheses testing.

In a hypotheses testing problem, a hypotheses of the form "$H : \theta$ is as M_θ" is a fuzzy hypotheses where M_θ is a membership function on the parameter space of θ.

For example, consider a study on the diameter of a factory manufacturing washers. If the mean diameter of washers conforms to the standard value $\mu_\circ$, we have in the classical case $H_\circ : \mu = \mu_\circ$ against $H_1 : \mu \neq \mu_\circ$. But it is clear that even if the average diameter of washers slightly differs from $\mu_\circ$ the washers are still acceptable and the production line is not considered non-standard. Hence, it is natural that if μ (mean actual and unknown for diameter washers) is almost $\mu_\circ$ the factory products are accepted and otherwise they are rejected. Thus, the true hypotheses is in this case are

$$\begin{aligned} \tilde{H}_\circ : \mu \quad & \text{is near to} \quad \mu_\circ \\ \tilde{H}_1 : \mu \quad & \text{is far from} \quad \mu_\circ. \end{aligned}$$

Now, $H_\circ$ and H_1 hypotheses can be modeled as fuzzy sets. We write the fuzzy null as $\tilde{H}_\circ : \mu = \tilde{\mu}_\circ$, where, $\tilde{\mu}_\circ$ is a fuzzy number. For simplicity we assume that $\tilde{\mu}_\circ$ is a triangular fuzzy number with α as left-width and right width [14,16].

3. Crisp rejection region and fuzzy test statistical

We introduce here a fuzzy test statistic which is preferred to the z-test statistic. In this regard we use hybrid numbers defined by Kaufman [10]. Hybrid numbers are a sum of random numbers and fuzzy numbers. We know that $\overline{X}_\circ = \overline{X} - \mu_\circ$, is a random number, which under $H_\circ$ has distribution of $N(0, \sigma^2/n)$. Now the number $\tilde{X}_H = \overline{X}_\circ + \tilde{\mu}_\circ$, which is the sum of the random number $\overline{X}_\circ$ and the fuzzy number $\tilde{\mu}_\circ$, and hence, $\tilde{X}_H$ is a hybrid number. In fact, $\tilde{X}_H$ is a combination of fuzzy null hypotheses with procise observation.

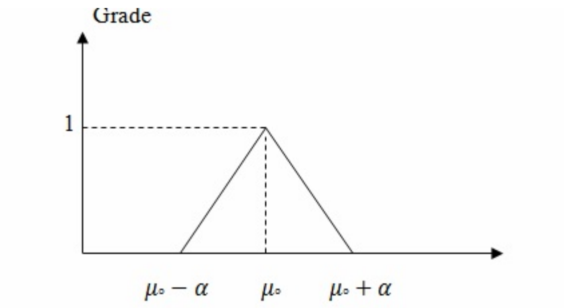

Figure 1: Triangular fuzzy number

Kato et al. [9], using the hybrid number($\tilde{X}_H$), defined the statistic $\tilde{Z}_H$ for testing the fuzzy hypotheses as follows:

$$\tilde{Z}_H = \frac{\tilde{X}_H - \mu_\circ}{\frac{\sigma}{\sqrt{n}}} = \frac{\overline{X}_\circ + \tilde{\mu}_\circ - \mu_\circ}{\frac{\sigma}{\sqrt{n}}} = \frac{\overline{X}_\circ}{\frac{\sigma}{\sqrt{n}}} + \frac{\tilde{\mu}_\circ - \mu_\circ}{\frac{\sigma}{\sqrt{n}}} = Z + (-\eta, 0, \eta),$$

where Z is the standard normal statistic, and $(-\eta, 0, \eta)$ is a triangular fuzzy number with $\eta = \frac{\alpha}{(\frac{\sigma}{\sqrt{n}})}$. Therefore, $\tilde{Z}_H$ is a hybrid number which may be used as a substitute for Z when using a fuzzy hypotheses test. To decide about the process, the value of $\tilde{Z}_H$ is compared with a crisp rejection region (Figure 2). If more than half of the area of $\tilde{Z}_H$ is in the rejection region then the null hypotheses is rejected, otherwise the null hypotheses is accepted.

Figure 2: Crisp rejection region

4. Fuzzy rejection region and fuzzy test statistic

When dealing with fuzzy characteristics by the classical time series method, it is possible that some information is lost. In this paper we test fuzzy hypotheses using fuzzy rejection regions and fuzzy statistics.

We define a fuzzy distance by a triangular fuzzy number, so that it is possible to use a fuzzy rejection region. In Figure 3, $\tilde{R}(Z)$ represents the fuzzy rejection region.

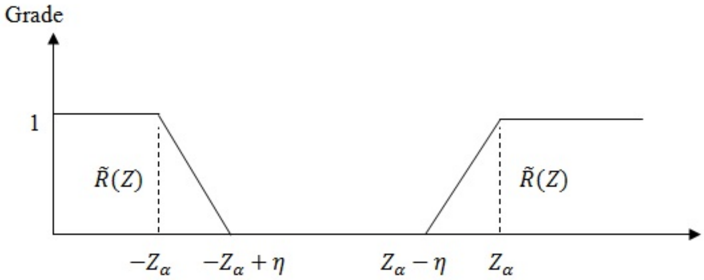

Figure 3: Fuzzy rejection region

The membership function of the fuzzy rejection region is defined as follows:

$$M_{\tilde{R}}(Z) = \begin{cases} \frac{-Z_\alpha+\eta-Z}{\eta}, & -Z_\alpha \leq Z \leq -Z_\alpha + \eta, \\ 0, & -Z_\alpha + \eta \leq Z \leq Z_\alpha - \eta, \\ \frac{Z-Z_\alpha+\eta}{\eta}, & Z_\alpha - \eta \leq Z \leq Z_\alpha, \\ 1, & (Z < -Z_\alpha) \vee (Z > Z_\alpha). \end{cases}$$

Here, decision for process depends on the degree that $\tilde{Z}_H$ is in the fuzzy rejection region[4].

We may define this degree in two ways.

First method: In this method, the decision-making for process depends on the area of the statistic $\tilde{Z}_H$ which is situated in the fuzzy rejection region. In Figure 4, the hatched region is the monitored area.

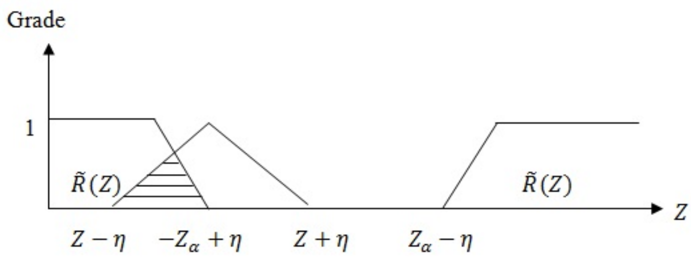

Figure 4: $\tilde{Z}_H$ statistic covers part of fuzzy rejection region

The degree of dependence on the rejection region is:

$$d_1 = \frac{\int \min(M_{\tilde{R}}(Z), M_{\tilde{Z}}(Z))dZ}{\int M_{\tilde{Z}}(Z)dZ}$$

If the $\tilde{Z}_H$ statistic is completely in the rejection region, then the degree of dependence is 1 ($d_1 = 1$) and the null hypotheses is rejected. If $\tilde{Z}_H$ is covered completely by the acceptance region, then the degree of dependence is zero $d_1 = 0$) resulting in the acceptance of the null hypotheses. When the amount of degree of dependence to fuzzy rejection region is between zero and one ($0 < d_1 < 1$), the null hypotheses is neither accepted nor rejected. The decision will depend on the amount of degree of dependence on the rejection region and a number β_1, which is a pre-determined either by a standard or by the quality control inspector.

Second method : In the second method, the quantity of degree of dependence on rejection region d_2, which is defined as the ratio of the length of the interval of statistic $\tilde{Z}_H$ located in fuzzy rejection region to the total length of the statistic. In figure 5, this is the ratio of AB to AC. If this value is equal to one,

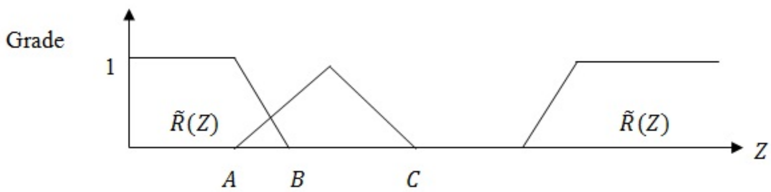

Figure 5: $\tilde{Z}_H$ statistic covers the part of fuzzy rejection region

then the null hypotheses is rejected, and if the value is equal to zero, then the null hypotheses is accepted. If the value of this ratio is between zero and one, the decision will be with quality control inspector who will compare the degree of dependence on the rejection region with a number β_2, which is a pre-determined either by a standard or by quality control inspector. If the degree of dependence on the rejection region is grater than β_2, then the null hypotheses is rejected and the process is considered "rather out of control" and if the expected value is less than β_2 then the null hypotheses is accepted, and the process is considered "rather in control". We have:

$$M_{\tilde{R}}(Z) = \begin{cases} 0, & (Z - \eta \geq -Z_\alpha + \eta) \wedge (Z + \eta \leq Z_\alpha - \eta), \\ 1 - \frac{Z + Z_\alpha}{2\eta}, & (Z - \eta < -Z_\alpha + \eta) \wedge (-Z_\alpha + \eta < Z + \eta < Z_\alpha + \eta), \\ 1 - \frac{-Z + Z_\alpha}{2\eta}, & (Z - \eta > -Z_\alpha + \eta) \wedge (Z + \eta > Z_\alpha - \eta), \\ 2 - \frac{Z_\alpha}{\eta}, & (Z - \eta < -Z_\alpha + \eta) \wedge (Z + \eta > Z_\alpha - \eta), \\ 1, & O.W. \end{cases}$$

5. A numerical Example

A sample of four units we taken on each of 20 consecutive days from a manufacturing process. The data is shown in table 1. If we know that the process follows a normal distribution with 8, and the process is in control if the mean is about 100, we have:

$$\begin{cases} \tilde{H}_\circ : \mu = \widetilde{100}, \\ \tilde{H}_\circ : \mu \neq \widetilde{100}. \end{cases}$$

If $\eta = 1$ and $Z_\alpha = 3$, then the fuzzy mean is:

$$\tilde{\mu}_\circ = (\mu_\circ - \eta\frac{\sigma}{\sqrt{n}}, \mu_\circ, \mu_\circ + \eta\frac{\sigma}{\sqrt{n}}) = (96, 100, 104)$$

In the fuzzy test comparison methods, d_1 is compared with $\beta_1 = 0.6$ and the d_2 with $\beta_2 = 0.5$. If d_1 is less than 0.6, then the process is pretty much in control, and if grater than 0.6, then the process is rather out of control. Also, when d_2 is less than 0.5 then the process is rather in control and the process is rather out of control if grater than 0.5. The value for d_1 and d_2 are displayed in Table 2.

Table 1. Data related to 20 samples from the process

Sample	X_1	X_2	X_3	X_4
1	93.335	100.317	104.281	105.738
2	96.408	102.725	101.664	109.418
3	100.205	103.556	105.408	106.729
4	94.766	98.873	107.352	91.486
5	106.503	100.479	109.322	92.303
6	98.124	83.141	101.652	104.669
7	108.023	100.272	101.425	96.77
8	113.747	99.549	93.468	99.297
9	97.191	106.073	81.224	109.464
10	94.439	105.869	105.714	111.429
11	98.66	97.632	98.984	86.384
12	109.35	89.921	109.781	110.869
13	93.171	89.032	91.75	98.541
14	115.566	114.463	107.644	99.217
15	108.043	117.346	98.128	113.902
16	108.93	117.74	99.134	114.55
17	107.417	109.925	113.74	114.752
18	120.711	105.88	111.711	108.227
19	102.164	104.814	123.525	121.066
20	98.791	121.621	120.281	120.758

Table 2. Results obtained from sampling

Sample	Comparison of $\tilde{Z}_H$ with crisp region	d_1	d_2	Comparison of $\tilde{Z}_H$ with fuzzy region
1	In control	1	1	In control
2	In control	1	1	In control
3	In control	1	1	In control
4	In control	1	1	In control
5	In control	1	1	In control
6	In control	1	1	In control
7	In control	1	1	In control
8	In control	1	1	In control
9	In control	1	1	In control
10	In control	0.998	0.955	Rather in control
11	In control	0.995	0.927	Rather in control
12	In control	0.985	0.877	Rather in control
13	In control	0.871	0.640	Rather in control
14	In control	0.574	0.347	Rather out of control
15	In control	0.552	0.331	Rather out of control
16	In control	0.421	0.239	Rather out of control
17	In control	0.131	0.068	Rather out of control
18	In control	0.090	0.046	Rather out of control
19	Out of control	0	0	Out of control
20	Out of control	0	0	Out of control

6. Conclusion

According to the results in Table 2, we see that in comparison method of fuzzy $\tilde{Z}_H$ test statistic with fuzzy acceptance region in comparison with defuzziffication acceptance region the level of degree of dependence to the acceptance region is precisely seen. And conclusion about the process based on this method; in addition to result in control and out of control, it has two results rather in control and approximately out of control. From the fourteenth sample, there are alarm signs but in the defuzziffication acceptance region in the nineteenth sample, we know that the process is out of control. Based on this method, the comparison of fuzzy test statistic with fuzzy acceptance region will be accurately concluded to the process. Of course, in the method of fuzzy test statistic comparison with fuzzy acceptance region two methods have been proposed.

Where, in the first method the value of degree of dependence is more accurately calculated than in the second method. But, the calculation of degree of dependence in the second method is easier than the first method.

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