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ON TOPOLOGICAL ENTROPY WITH THE LEVELS (α, β) OF $\alpha\beta$-RELATIVE DYNAMICAL SYSTEMS

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Abstract. In this paper, a relative intuitionistic dynamical system with the levels (α, β), as a mathematical model compatible with a natural phenomenon, is proposed. In addition, the notion of RI topological entropy with the levels (α, β) for RI dynamical systems with the levels (α, β) is defined and its properties are studied. As a significant result, it was shown that, this topological entropy is an invariant object up to conjugate relation.

Keywords: Relative intuitionistic topological entropy, Relative intuitionistic dynamical system, Dynamical system, Topological entropy.
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1. Introduction

After defining the concept of measure entropy in ergodic theory given by Kolmogorov [10] and Sinai [16], Adler et al. [1] introduced the topological entropy of continuous maps for a compact dynamical system in general. Let X be a compact topological space and U be an open cover of X. The topological entropy of U was defined by $H(U) = \log N(U)$, where $N(U)$ is the minimal cardinality of a sub-cover of U. For any two open covers U and V of X, the join refinement was proposed by $U \vee V = \{u \cap v : u \in U, v \in V\}$. If f is a continuous map, $(X; f)$ is called a discrete dynamical system and the topological entropy of f with respect to the open cover U is defined in [1] by:

$$h(U, f) = \lim_{n\to\infty} \frac{1}{n} H(\bigvee_{i=0}^{n-1} f^{-i}U)$$

And the topological entropy of f was introduced in [1] by :

$$h(f) = \sup\{h(f, U) : U \text{ is an open cover of } f.$$

Since the fuzzy set theory was defined by Zadeh in 1965 [18], this notion has been widely used in many different fields [5, 8]. Molaei [12] defined a topology from an observer's viewpoint by considering a fuzzy set as an observer. This model was further developed and studied in a lot of research [13, 14]. Subsequently, based on this topology, $(\alpha, \mu)-$topology and relative dynamical

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systems with the level α were proposed and some properties of these concepts were investigated [13].
Intuitionistic fuzzy set (IFS) as a generalization of Zadeh's fuzzy set was defined by Atanassov [2, 3]. Let X be a non-empty set, an intuitionistic fuzzy set (IFS in short) A in X is an object which has the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

which was characterized by membership function $\mu_A : X \longrightarrow [0,1]$, non-membership function $\nu_A : X \longrightarrow [0,1]$, and the degree of hesitancy $\pi(x)$. Note that, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and $\pi(x) = 1 - \mu_A(x) - \nu_A(x)$ for every $x \in X$.

In recent years, many scholars from different fields, such as decision making, medical diagnosis, logic programming, pattern recognition, market prediction and machine learning, etc., have become interested in IFS [6, 10]. Because in these fields, prediction always includes some degrees of uncertainty, the intuitionistic fuzzy set is a suitable mathematical model for them. In mathematics, various concepts of fuzzy mathematics have been generalized to intuitionistic fuzzy sets [4, 15].

One of the main goals of this study is to extend the notion relating to topological entropy with the level α to relative intuitionistic topological entropy (RIT in short) with the levels (α, β). The other main goal is to generalize the relative dynamical system with the level α to the relative intuitionistic (RI in short) dynamical system with the levels (α, β). Consequently, at first basic notions are depicted and then concepts of $(\alpha\beta, \langle\mu, \nu\rangle)-$ topology, RIT entropy with the levels(α, β), and also RI entropy on RI dynamical system with the levels (α, β) are introduced. Some properties of these notions are investigated and it is proved that this entropy is also a topological invariant according to conjugation. In the final part of this article, an example of calculating RIT entropy with the levels(α, β) on RI dynamical system with the levels (α, β) is provided.

2. Relative intuitionistic topological entropy with the level (α, β)

This section starts with some basic notions of intuitionistic fuzzy sets, and then continues with introducing $(\alpha\beta, \langle\mu, \nu\rangle)-$ topology space and also relative intuitionistic topological entropy with the levels (α, β) while proving their properties

Definition 2.1 (3)**.** Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ and
$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in X\}$ be intuitionistic fuzzy sets. Then:
(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $A^C = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$,
(d) $A \cap B = \{\langle x, \inf\{\mu_A(x), \mu_B(x)\}, \sup\{\nu_A(x), \nu_B(x)\}\rangle | x \in X\}$,

(e) $A \cup B = \{\langle x, \sup\{\mu_A(x), \mu_B(x)\}, \inf\{\nu_A(x), \nu_B(x)\}\rangle | x \in X\}$.

The IF sets $\chi_\emptyset = 0_\sim = \{\langle x, 0, 1\rangle\rangle | x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle | x \in X\}$ are the empty set and the whole set X, respectively.

In this paper, we shall use the notation $A = \langle \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$.

The next proposition presents some properties of intuitionistic fuzzy sets.

Proposition 2.2. *Let A, $A_i\,(i \in I)$ be IFSs in X and B, $B_j\,(j \in J)$ be IFSs in Y and $f : X \to Y$ be a map then:*
i)$f^{-1}(\underset{j}{\cup} B_j) = \underset{j}{\cup} f^{-1}(B_j)$,
ii) $f^{-1}(\underset{j}{\cap} B_j) = \underset{j}{\cap} f^{-1}(B_j)$,
iii) $f^{-1}(1_\sim) = 1_\sim,\ f^{-1}(0_\sim) = 0_\sim$,
iv) $f(\underset{i}{\cup} A_i) = \underset{i}{\cup} f(A_i)$,
v) $f^{-1}(\bar{B}) = \overline{f^{-1}(B)}$.

Proof. Proofs will be found in [7]. □

The set of all intuitionistic fuzzy sets in X denoted by IFS(X) and an IF set $A = \langle \mu_A, \nu_A\rangle$ is called a relative intuitionistic observer (RIO in short) of set X.

Definition 2.3. Let $A = \langle \mu_A, \nu_A\rangle$ be a RIO of X. Then a relative intuitionistic topology (RIT in short) on X is a collection $\tau_{(\mu_A, \nu_A)}$ of subsets of $A = \langle \mu_A, \nu_A\rangle$ satisfying the following axioms:
(i) $\chi_\emptyset$ and $\langle \mu_A, \nu_A\rangle \in \tau_{(\mu_A, \nu_A)}$,
(ii) $G_1 \cap G_2 \in \tau_{(\mu_A, \nu_A)}$ for any $G_1, G_2 \in \tau_{(\mu_A, \nu_A)}$,
(iii) $\underset{i \in I}{\cup} G_i \in \tau_{(\mu_A, \nu_A)}$ for any family $\{G_i | i \in I\} \subseteq \tau_{(\mu_A, \nu_A)}$.

The pair $(X, \tau_{(\mu_A, \nu_A)})$ is called a relative intuitionistic topology space (RITS in short) and the elements of $\tau_{(\mu_A, \nu_A)}$ are called $\langle \mu_A, \nu_A\rangle$- open observer. We say $B = \langle \mu_B, \nu_B\rangle$ is $\langle \mu_A, \nu_A\rangle$- closed if $B^C = \langle \nu_B, \mu_B\rangle \in \tau_{(\mu_A, \nu_A)}$ [9] .

Theorem 2.4. *Let $(X, \tau_{(\mu_A, \nu_A)})$, be a RIT. If $f : Y \to X$ is a map and $\tau_{(\mu_A \circ f, \nu_A \circ f)} = \{(\mu_i \circ f, \nu_i \circ f) :\ \langle \mu_i, \nu_i\rangle \in \tau_{(\mu_A, \nu_A)}\}$, then $(Y, \tau_{(\mu_A \circ f, \nu_A \circ f)})$ is a RIT.*

Proof. The proof will be found in [9] . □

In the next definition, notion of $(\alpha\beta, \langle \mu, \nu\rangle)-$ topology which is a basic concept will be given.

Definition 2.5. Let $(X, \tau_{(\mu, \nu)})$ be a RIT space, $\langle \mu_i, \nu_i\rangle \in \tau_{(\mu, \nu)}$, $\langle \mu_i, \nu_i\rangle_{\alpha\beta} = \{x \in X : \mu_i(x) > \alpha, \nu_i(x) < \beta, \alpha + \beta \leq 1\}$.

Collection $(\tau_{(\mu,\nu)})_{\alpha\beta} = \{\langle \mu_i, \nu_i\rangle_{\alpha\beta} : \langle \mu_i, \nu_i\rangle \in \tau_{(\mu,\nu)}\}$ is called $(\alpha\beta, \langle \mu, \nu\rangle)-$ topology if $\langle \mu_i, \nu_i\rangle \in \tau_{(\mu,\nu)}, i \in I$, then we have $\underset{i\in I}{\cup}\langle \mu_i, \nu_i\rangle_{\alpha\beta} = \langle \underset{i\in I}{\cup}\langle \mu_i, \nu_i\rangle\rangle_{\alpha\beta}$.

Theorem 2.6. *$(\langle \mu, \nu\rangle_{\alpha\beta}, (\tau_{(\mu,\nu)})_{\alpha\beta})$ is a topological space.*

Proof.
(i) $\chi_\emptyset = \langle \chi_\emptyset\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$ and also $\langle \mu, \nu\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$.
(ii) if $\langle \mu_1, \nu_1\rangle_{\alpha\beta}$ and $\langle \mu_2, \nu_2\rangle_{\alpha\beta}$ belong to $(\tau_{(\mu,\nu)})_{\alpha\beta}$ then $\langle \mu_1, \nu_1\rangle_{\alpha\beta} \cap \langle \mu_2, \nu_2\rangle_{\alpha\beta} = \langle \mu_1 \wedge \mu_2, \nu_1 \vee \nu_2\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$.
(iii) if $\langle \mu_i, \nu_i\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}, i \in I$, then by definition $\underset{i\in I}{\cup}\langle \mu_i, \nu_i\rangle_{\alpha\beta} = \langle \underset{i\in I}{\cup}\langle \mu_i, \nu_i\rangle\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$. □

X is called an $(\alpha\beta, \langle \mu, \nu\rangle)-$ topology space if $(\langle \mu, \nu\rangle_{\alpha\beta}, (\tau_{(\mu,\nu)})_{\alpha\beta})$ is an $(\alpha\beta, \langle \mu, \nu\rangle)-$ topology space. $\langle \mu_i, \nu_i\rangle_{\alpha\beta}$ is called $(\alpha\beta, \langle \mu, \nu\rangle)-$ open if $\langle \mu_i, \nu_i\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$ and collection $\{\langle \mu_i, \nu_i\rangle_{\alpha\beta} : \langle \mu_i, \nu_i\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}, i = 1, ..., n\}$ is called $(\alpha\beta, \langle \mu, \nu\rangle)-$ open cover if $\overset{n}{\underset{i=1}{\cup}}\langle \mu_i, \nu_i\rangle_{\alpha\beta} = \langle \mu, \nu\rangle_{\alpha\beta}$.

In defining topological entropy, the function needed to be continuous. Now we prepare the equivalent definition that is $\alpha\beta$- relative continuous.

Now we prepare the equivalent definition that is - relative continuous

Definition 2.7. Let X and Y be $(\alpha\beta, \langle \mu_A, \nu_A\rangle)$ and $(\alpha\beta, \langle \mu_B, \nu_B\rangle)-$ topology spaces, respectively. $f : X \to Y$ is said to be a $\alpha\beta$- relative continuous map if $\langle \mu_i \circ f, \nu_i \circ f\rangle_{\alpha\beta} \cap \langle \mu_A, \nu_A\rangle_{\alpha\beta} = f^{-1}(\langle \mu_i, \nu_i\rangle_{\alpha\beta}) \cap \langle \mu_A, \nu_A\rangle_{\alpha\beta} \in (\tau_{(\mu_A,\nu_A)})_{\alpha\beta}$ for every $\langle \mu_i, \nu_i\rangle_{\alpha\beta} \in (\tau_{(\mu_B,\nu_B)})_{\alpha\beta}$.
f is $\alpha\beta$- relative homeomorphism if f is a bijection and both f and f^{-1} are $\alpha\beta$- relative continuous map.

Proposition 2.8. *Let $f : X \to Y$ be a $\alpha\beta$- relative continuous map. Then*
(i) $\langle \mu^i, \nu^i\rangle_{\alpha\beta} \cap \langle \mu^j, \nu^j\rangle_{\alpha\beta} = \langle\langle \mu^i, \nu^i\rangle \cap \langle \mu^j, \nu^j\rangle\rangle_{\alpha\beta}$,
ii) $f^{-1}(\langle \mu, \nu\rangle_{\alpha\beta}) = \langle f^{-1}\langle \mu, \nu\rangle\rangle_{\alpha\beta}$.

Proof.
(i) $x \in \langle \mu^i, \nu^i\rangle_{\alpha\beta} \cap \langle \mu^j, \nu^j\rangle_{\alpha\beta}$ iff $\mu^i(x) > \alpha, \nu^i(x) < \beta$ and $\mu^j(x) > \alpha, \nu^j(x) < \beta$ iff $\inf\{\mu^i(x), \mu^j(x)\} > \alpha$ and $\sup\{\nu^i(x), \nu^j(x)\} < \beta$ iff $x \in \langle\langle \mu^i, \nu^i\rangle \cap \langle \mu^j, \nu^j\rangle\rangle_{\alpha\beta}$.
ii) $f^{-1}(\langle \mu_i, \nu_i\rangle_{\alpha\beta}) = \langle \mu_i \circ f, \nu_i \circ f\rangle_{\alpha\beta} = \langle f^{-1}\langle \mu, \nu\rangle\rangle_{\alpha\beta}$. □

Definition 2.9. X is called $(\alpha\beta, \langle \mu, \nu\rangle)-$ Hausdorff space if $(\langle \mu, \nu\rangle_{\alpha\beta}, (\tau_{(\mu,\nu)})_{\alpha\beta})$ is a Hausdorff space, and it is called $(\alpha\beta, \langle \mu, \nu\rangle)-$ compact if $(\langle \mu, \nu\rangle_{\alpha\beta}, (\tau_{(\mu,\nu)})_{\alpha\beta})$ is a compact space.

In topological spaces, continuous functions have specific features. In the next theorem, we characterize some equivalent features for $\alpha\beta$- relative continuous maps. It is proved that every map between $(\alpha\beta, \langle \mu, \nu\rangle)-$ topology spaces is an $\alpha\beta$- relative continuous map and every $\alpha\beta$- relative continuous map carries out $(\alpha\beta, \langle \mu, \nu\rangle)-$ Hausdorff space to $(\alpha\beta, \langle \mu, \nu\rangle)-$ Hausdorff space.

Theorem 2.10. *Let $f : Y \to X$ be a map and X be a $(\alpha\beta, \langle \mu, \nu \rangle)-$ topology space then:*
(i) *Y is an $(\alpha\beta, \langle \mu \circ f, \nu \circ f \rangle)-$ topology space,*
(ii) *f is an $\alpha\beta$- relative continuous map,*
(iii) *if X is an $(\alpha\beta, \langle \mu, \nu \rangle)-$ Hausdorff space, then Y is a $(\alpha\beta, \langle \mu \circ f, \nu \circ f \rangle)-$ Hausdorff space.*

Proof.
(i) By Theorem 2.4, $(Y, \tau_{(\mu \circ f, \nu \circ f)})$ is a RIT space.

Let $(\tau_{(\mu \circ f, \nu \circ f)})_{\alpha\beta} = \{\langle \mu_i \circ f, \nu_i \circ f \rangle_{\alpha\beta} : \langle \mu_i, \nu_i \rangle \in \tau_{(\mu,\nu)}\}$ and for $i \in I$, $\langle \mu_i \circ f, \nu_i \circ f \rangle_{\alpha\beta} \in (\tau_{(\mu \circ f, \nu \circ f)})_{\alpha\beta}$.

We will prove that $\underset{i \in I}{\cup} \langle \mu_i \circ f, \nu_i \circ f \rangle_{\alpha\beta} = \langle \underset{i \in I}{\cup} \langle \mu_i \circ f, \nu_i \circ f \rangle \rangle_{\alpha\beta}$.

For any $i \in I$, $\langle \mu_i \circ f, \nu_i \circ f \rangle_{\alpha\beta} \subset \langle \underset{i \in I}{\cup} \langle \mu_i \circ f, \nu_i \circ f \rangle \rangle_{\alpha\beta}$.
On the other hand, if $y \in \langle \underset{i \in I}{\cup} \langle \mu_i \circ f, \nu_i \circ f \rangle \rangle_{\alpha\beta}$ then $(\underset{i \in I}{\vee} \mu_i)(f(y)) = (\underset{i \in I}{\vee} \mu_i \circ f)(y) > \alpha$ and $(\underset{i \in I}{\wedge} \nu_i)(f(y)) = (\underset{i \in I}{\wedge} \nu_i \circ f)(y) < \beta$. Thus $f(y) \in \langle \underset{i \in I}{\cup} \langle \mu_i, \nu_i \rangle \rangle_{\alpha\beta} = \underset{i \in I}{\cup} \langle \mu_i, \nu_i \rangle_{\alpha\beta}$ and therefore $\exists i \in I$ such that $\mu_i(f(y)) > \alpha$ and $\nu_i(f(y)) < \beta$. This implies $y \in \underset{i \in I}{\cup} \langle \mu_i \circ f, \nu_i \circ f \rangle_{\alpha\beta}$.

(ii) $f^{-1}(\langle \mu_i, \nu_i \rangle_{\alpha\beta}) \cap \langle \mu \circ f, \nu \circ f \rangle_{\alpha\beta} = \langle f^{-1}(\langle \mu_i, \nu_i \rangle) \cap \langle \mu \circ f, \nu \circ f \rangle \rangle_{\alpha\beta} \in (\tau_{(\mu \circ f, \nu \circ f)})_{\alpha\beta}$.

(iii) Let $y_1, y_2 \in \langle \mu \circ f, \nu \circ f \rangle_{\alpha\beta}$. $\mu(f(y_i)) > \alpha, i = 1, 2$ and $\nu(f(y_i)) < \beta, i = 1, 2$. Therefore, $f(y_i) \in \langle \mu, \nu \rangle_{\alpha\beta}, i = 1, 2$.
Since X is an $(\alpha\beta, \langle \mu, \nu \rangle)-$ Hausdorff space, there exist $\langle \mu_1, \nu_1 \rangle_{\alpha\beta}$ and $\langle \mu_2, \nu_2 \rangle_{\alpha\beta}$ belonging to $(\tau_{(\mu,\nu)})_{\alpha\beta}$ such that $f(y_i) \in \langle \mu_i, \nu_i \rangle_{\alpha\beta}, i = 1, 2$ and $\overset{2}{\underset{i=1}{\cap}} \langle \mu_i, \nu_i \rangle_{\alpha\beta} = \emptyset$.
f is an $\alpha\beta$- relative continuous map thus $f^{-1}(\langle \mu_i, \nu_i \rangle_{\alpha\beta}) \cap \langle \mu \circ f, \nu \circ f \rangle_{\alpha\beta} \in (\tau_{(\mu \circ f, \nu \circ f)})_{\alpha\beta}$ and $y_i \in f^{-1}(\langle \mu_i, \nu_i \rangle_{\alpha\beta}) \cap \langle \mu \circ f, \nu \circ f \rangle_{\alpha\beta}$.

Now it will be proved that $\overset{2}{\underset{i=1}{\cap}} (f^{-1}(\langle \mu_i, \nu_i \rangle_{\alpha\beta}) \cap \langle \mu \circ f, \nu \circ f \rangle_{\alpha\beta}) = \emptyset$. If $t \in \overset{2}{\underset{i=1}{\cap}} (f^{-1}(\langle \mu_i, \nu_i \rangle_{\alpha\beta}) \cap \langle \mu \circ f, \nu \circ f \rangle_{\alpha\beta})$, then $\mu_i(f(t)) > \alpha$ and $\nu_i(f(t)) < \beta$ for $i = 1, 2$ and this implies that $f(t) \in \overset{2}{\underset{i=1}{\cap}} \langle \mu_i, \nu_i \rangle_{\alpha\beta}$, which is a contradiction. □

The next theorem determine the situation of $(\alpha\beta, \langle \mu, \nu \rangle)-$ open covers when f is a $\alpha\beta$- relative continuous map.

Theorem 2.11. *Let X be an $(\alpha\beta, \langle\mu,\nu\rangle)-$ topology space and $f : X \to X$ be an $\alpha\beta$- relative continuous map.*
Moreover, let $\theta = \{\langle\mu_1,\nu_1\rangle_{\alpha\beta}, \langle\mu_2,\nu_2\rangle_{\alpha\beta}, ..., \langle\mu_n,\nu_n\rangle_{\alpha\beta}\}$ be an $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover for $\langle\mu,\nu\rangle_{\alpha\beta}$.
Then $f^{-1}(\theta) = \{\langle\langle\mu,\nu\rangle \cap f^{-1}(\langle\mu_1,\nu_1\rangle)\rangle_{\alpha\beta}, \langle\langle\mu,\nu\rangle \cap f^{-1}(\langle\mu_2,\nu_2\rangle)\rangle_{\alpha\beta}, ..., \langle\langle\mu,\nu\rangle \cap f^{-1}(\langle\mu_n,\nu_n\rangle)\rangle_{\alpha\beta}\}$ is an $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover for $\langle\langle\mu,\nu\rangle \cap f^{-1}(\langle\mu,\nu\rangle)\rangle_{\alpha\beta}$.

Proof. Since f is an $\alpha\beta$- relative continuous map, we have $f^{-1}(\langle\mu_i,\nu_i\rangle_{\alpha\beta}) \cap \langle\mu,\nu\rangle_{\alpha\beta} = \langle f^{-1}(\langle\mu_i,\nu_i\rangle) \cap \langle\mu,\nu\rangle\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$.
Now if $t \in \langle\langle\mu,\nu\rangle \cap f^{-1}(\langle\mu,\nu\rangle)\rangle_{\alpha\beta}$ then $\mu(t) \wedge \mu(f(t)) > \alpha$ and $\nu(t) \vee \nu(f(t)) < \beta$, so $f(t) \in \langle\mu,\nu\rangle_{\alpha\beta}$.
Since θ is an $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover for $\langle\mu,\nu\rangle_{\alpha\beta}$, there is $1 \le m \le n$ such that $f(t) \in \langle\mu_m,\nu_m\rangle_{\alpha\beta}$.
Thus $\mu_m(f(t)) > \alpha$ and $\nu_m(f(t)) < \beta$ and these imply $t \in \langle f^{-1}(\langle\mu_m,\nu_m\rangle)\rangle_{\alpha\beta} = f^{-1}(\langle\mu_m,\nu_m\rangle_{\alpha\beta})$.
Also, we have $t \in \langle\mu,\nu\rangle_{\alpha\beta}$. Therefore, $t \in \langle\langle\mu,\nu\rangle \cap f^{-1}(\langle\mu,\nu\rangle)\rangle_{\alpha\beta}$.
Now we can conclude $f^{-1}(\theta)$ is an $(\alpha\beta, \langle\mu,\nu\rangle)$- open cover for

$$\langle\langle\mu,\nu\rangle \cap f^{-1}(\langle\mu,\nu\rangle)\rangle_{\alpha\beta}$$

. □

Let $\theta = \{\langle\mu_1,\nu_1\rangle_{\alpha\beta}, \langle\mu_2,\nu_2\rangle_{\alpha\beta}, ..., \langle\mu_n,\nu_n\rangle_{\alpha\beta}\}$ be an $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover for $\langle\mu,\nu\rangle_{\alpha\beta}$. $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover σ is called a subcover if $\sigma \subseteq \theta$.

Definition 2.12. Let X be an $(\alpha\beta, \langle\mu,\nu\rangle)$- compact space, θ be an $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover for $\langle\mu,\nu\rangle_{\alpha\beta}$ and $N(\theta)$ denotes the number of set elements that is a finite subcover of θ with the smallest cardinality.
The relative intuitionistic topological entropy of the cover θ with the levels (α,β) is defined by:

$$H_{\alpha\beta}(\theta) := \log N(\theta).$$

Proposition 2.13. *Let X be an $(\alpha\beta, \langle\mu,\nu\rangle)$- compact space, θ be an $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover for $\langle\mu,\nu\rangle_{\alpha\beta}$. Then*
(i) $H_{\alpha\beta}(\theta) \ge 0$,

ii) $H_{\alpha\beta}(\theta) = 0$ *iff* $N(\theta) = 1$ *iff* $\langle\mu,\nu\rangle_{\alpha\beta} \in \theta$.

Proof. Proofs are obvious. □

Definition 2.14. An $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover λ is a refinement of $(\alpha\beta, \langle\mu,\nu\rangle)-$ open cover θ, shown by $\theta \prec \lambda$, if every member of λ is a subset of a member of θ.

The next theorem proves that if a partition becomes finer, then its entropy becomes larger.

Theorem 2.15. *If $\theta \prec \lambda$ then $H_{\alpha\beta}(\theta) \le H_{\alpha\beta}(\lambda)$.*

Proof. Let $\{\langle\mu_\lambda^i, \nu_\lambda^i\rangle_{\alpha\beta} : i = 1, ..., N(\sigma)\}$ be a subcover of λ with the smallest cardinality. For any $i \in \{1, ..., N(\sigma)\}$ there exists $\langle\mu_\theta^i, \nu_\theta^i\rangle_{\alpha\beta} \in \theta$ such that $\langle\mu_\lambda^i, \nu_\lambda^i\rangle_{\alpha\beta} \subseteq \langle\mu_\theta^i, \nu_\theta^i\rangle_{\alpha\beta}$.
Therefore $\{\langle\mu_\theta^i, \nu_\theta^i\rangle_{\alpha\beta} : i = 1, ..., N(\sigma)\}$ covers $\langle\mu, \nu\rangle_{\alpha\beta}$ and is a subcover for θ. Now we can conclude $N(\theta) \leq N(\lambda)$. □

Theorem 2.16. *If $f : X \to X$ be an $\alpha\beta$- relative continuous map, and θ is a $(\alpha\beta, \langle\mu, \nu\rangle)-$ open cover then $H(f^{-1}(\theta)) \leq H(\theta)$.*

Proof. If $\acute{\theta} = \{\langle\mu^i, \nu^i\rangle_{\alpha\beta} : i = 1, ..., N(\theta)\}$ is a subcover of θ with the smallest cardinality, then $f^{-1}(\acute{\theta}) = \{\langle\mu, \nu\rangle_{\alpha\beta} \cap f^{-1}(\langle\mu^i, \nu^i\rangle_{\alpha\beta}) : i = 1, ..., N(\theta)\}$ is a $(\alpha\beta, \langle\mu, \nu\rangle)-$ open cover for $\langle\mu, \nu\rangle_{\alpha\beta} \cap f^{-1}(\langle\mu, \nu\rangle_{\alpha\beta})$. Hence $N(f^{-1}(\theta)) \leq N(f^{-1}(\acute{\theta})) \leq N(\theta)$. □

From above theorem one obtains:

Corollary 2.17. *If $f : X \to X$ is an $\alpha\beta$- relative homeomorphism and θ is a $(\alpha\beta, \langle\mu, \nu\rangle)-$ open cover, then $H_{\alpha\beta}(\theta) = H_{\alpha\beta}(f^{-1}(\theta))$.*

Proof. $H(f^{-1}(\theta)) \leq H(\theta) = H(f(f^{-1}(\theta))) \leq H(f^{-1}(\theta))$. □

The join refinement of two partitions is a basic concept for defining the entropy of a dynamical system. In the next definition, the notion of join refinement of two $(\alpha\beta, \langle\mu, \nu\rangle)-$ open covers is introduced.

Definition 2.18. Let $\theta = \{\langle\mu^i, \nu^i\rangle_{\alpha\beta} : i \in I, \langle\mu^i, \nu^i\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta})\}$ and $\sigma = \{\langle\mu^j, \nu^j\rangle : j \in J, \langle\mu^j, \nu^j\rangle \in (\tau_{(\mu,\nu)})_{\alpha\beta})\}$ be two $(\alpha\beta, \langle\mu, \nu\rangle)-$ open covers. The join refinement of θ and σ is defined by:

$$\theta \vee \sigma := \{\langle\mu^i, \nu^i\rangle_{\alpha\beta} \cap \langle\mu^j, \nu^j\rangle_{\alpha\beta} : \langle\mu^i, \nu^i\rangle_{\alpha\beta} \in \theta, \langle\mu^j, \nu^j\rangle_{\alpha\beta} \in \sigma\}.$$

Theorem 2.19. *If $\theta = \{\langle\mu^i, \nu^i\rangle_{\alpha\beta} : i \in I, \langle\mu^i, \nu^i\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}\}$ and $\sigma = \{\langle\mu^j, \nu^j\rangle : j \in J, \langle\mu^j, \nu^j\rangle \in (\tau_{(\mu,\nu)})_{\alpha\beta}\}$ are two $(\alpha\beta, \langle\mu, \nu\rangle)-$ open covers, then $\theta \vee \sigma$ is an $(\alpha\beta, \langle\mu, \nu\rangle)-$ open cover for $\langle\mu, \nu\rangle_{\alpha\beta}$.*

Proof.

$$\underset{i\in I j\in J}{\cup}(\langle\mu^i, \nu^i\rangle_{\alpha\beta} \cap \langle\mu^j, \nu^j\rangle_{\alpha\beta}) = (\underset{i\in I}{\cup}\langle\mu^i, \nu^i\rangle_{\alpha\beta}) \cap (\underset{j\in J}{\cup}\langle\mu^j, \nu^j\rangle)_{\alpha\beta} = \langle\mu, \nu\rangle_{\alpha\beta}.$$

□

Now let us present the main theorem of this section, which helps us define the entropy of a dynamical system.

Theorem 2.20. *If $\theta = \{\langle\mu^i, \nu^i\rangle_{\alpha\beta} : i \in I, \langle\mu^i, \nu^i\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}\}$ and $\sigma = \{\langle\mu^j, \nu^j\rangle : j \in J, \langle\mu^j, \nu^j\rangle \in (\tau_{(\mu,\nu)})_{\alpha\beta})\}$ are two $(\alpha\beta, \langle\mu, \nu\rangle)-$ open covers, then $H_{\alpha\beta}(\theta \vee \sigma) \leq H_{\alpha\beta}(\theta) + H_{\alpha\beta}(\sigma)$.*

Proof. Let $\acute{\theta} \subseteq \theta, \acute{\sigma} \subseteq \sigma$ be subcovers of θ and σ with the smallest cardinality m and n. Thus $H(\theta) = \log m$ and $H(\sigma) = \log n$. $\acute{\theta} \vee \acute{\sigma}$ is a subcover of $\theta \vee \sigma$, so $H(\theta \vee \sigma) \leq \log nm = \log n + \log m = H(\theta) + H(\sigma)$. □

3. **A relative intuitionistic topological entropy on a relative intuitionistic dynamical system with the levels** (α, β)

In this section a relative intuitionistic topological entropy on a relative intuitionistic dynamical system with the levels (α, β) is introduced and it is proved that this topological entropy is an invariant object up to conjugate relation.

Definition 3.1. Let X be an $(\alpha\beta, \langle\mu, \nu\rangle)-$ compact space and $f : X \to X$ be an $\alpha\beta$- relative continuous map. $(X, (\tau_{(\mu,\nu)})_{\alpha\beta}, f)$ is called a relative intuitionistic dynamical system with the levels (α, β).

Theorem 3.2. *If* $\theta = \{\langle\mu^i, \nu^i\rangle_{\alpha\beta} : i \in I, \langle\mu^i, \nu^i\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}\}$ *and* $\sigma = \{\langle\mu^j, \nu^j\rangle_{\alpha\beta} : j \in J, \langle\mu^j, \nu^j\rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta})\}$ *is two* $(\alpha\beta, \langle\mu, \nu\rangle)-$ *open cover then:*
(i) $f^{-1}(\theta \vee \sigma) = f^{-1}(\theta) \vee f^{-1}(\sigma)$,
(ii) $H_{\alpha\beta}(f^{-1}(\theta \vee \sigma)) = H_{\alpha\beta}(f^{-1}(\theta) \vee f^{-1}(\sigma))$.

Proof.
(i) $f^{-1}(\langle\mu^i, \nu^i\rangle_{\alpha\beta} \cap \langle\mu^j, \nu^j\rangle_{\alpha\beta}) \cap \langle\mu, \nu\rangle_{\alpha\beta} = f^{-1}(\langle\mu^i, \nu^i\rangle_{\alpha\beta}) \cap f^{-1}(\langle\mu^j, \nu^j\rangle_{\alpha\beta}) \cap \langle\mu, \nu\rangle_{\alpha\beta}$. □

Theorem 3.3. *Let* $\{(a_i)\}_{i=1}^{\infty}$ *be a sequence of nonnegative numbers such that* $a_{r+s} \leq a_r + a_s$ *for each* $r, s = 1, 2, \ldots$ *, then* $\lim\limits_{n\to\infty} \frac{1}{n} a_n$ *exists.*

Proof. The proof can be found in [14]. □

Theorem 3.4. *Let* $(X, (\tau_{(\mu,\nu)})_{\alpha\beta}, f)$ *be a relative intuitionistic dynamical system with the level* (α, β)*, and* θ *be an* $(\alpha\beta, \langle\mu, \nu\rangle)$ *-open cover then*

$$h_{\alpha\beta}(f, \theta) := \lim_{n\to\infty} \frac{1}{n} H_{\alpha\beta}(\bigvee_{i=0}^{n-1} f^{-i}\theta)$$

exists.

Proof. Let $x_n = H(\bigvee_{i=0}^{n-1} f^{-i}\theta)$. For every $m, n \in N$ we have:
$x_{n+m} = H(\bigvee_{i=0}^{m+n-1} f^{-i}\theta) \leq H(\bigvee_{i=0}^{m-1} f^{-i}\theta) + H(f^{-m}(\bigvee_{i=0}^{n-1} f^{-i}\theta)) \leq H(\bigvee_{i=0}^{m-1} f^{i}\theta) + H(\bigvee_{i=0}^{n-1} f^{-i}\theta) = x_m + x_n$. Thus $\{x_n\}_{n\in N}$ is a sub additive sequence so $\lim\limits_{n\to\infty} \frac{1}{n} x_n$ exists. □

According to the previous theorem, the following definition is well defined.

Definition 3.5. $h_{\alpha\beta}(f) = \sup\{h_{\alpha\beta}(f, \theta) : \theta$ is an $(\alpha\beta, \langle\mu, \nu\rangle)-$ open cover$\}$ called RIT entropy with the level (α, β) of f.

Definition 3.6. Let X be $(\alpha\beta, \langle\mu, \nu\rangle)-$ compact space. Two $\alpha\beta$-relative continuous maps f and g are said $\langle\mu,\nu\rangle_{\alpha\beta}$- conjugate if there exists an $\alpha\beta$- relative homeomorphism $\varphi : (X, (\tau_{(\mu,\nu)})_{\alpha\beta} \to (X, (\tau_{(\mu,\nu)})_{\alpha\beta}$ such that $\varphi \circ f = g \circ \varphi$.

The next theorem implies that the RIT entropy with the level (α, β) is an invariant object under $\langle\mu_A, \nu_A\rangle_{\alpha\beta}$-conjugate relations.

Theorem 3.7. *Let X be an $(\alpha\beta,\langle\mu,\nu\rangle)$-compact space. If $f:(X,(\tau_{(\mu,\nu)})_{\alpha\beta})\to(X,(\tau_{(\mu,\nu)})_{\alpha\beta})$ and $g:(X,(\tau_{(\mu,\nu)})_{\alpha\beta})\to(X,(\tau_{(\mu,\nu)})_{\alpha\beta})$ are $\langle\mu_A,\nu_A\rangle_{\alpha\beta}$-conjugate, then $h_{\alpha\beta}(f)=h_{\alpha\beta}(g)$.*

Proof. Let α be a RIO cover for $\langle\mu_A,\nu_A\rangle$. $h(g,\alpha)=\limsup\limits_{n\to\infty}\frac{1}{n}H(\bigvee\limits_{i=0}^{n-1}g^{-i}\alpha)=\limsup\limits_{n\to\infty}\frac{1}{n}H(\varphi^{-1}(\bigvee\limits_{i=0}^{n-1}g^{-i}\alpha))=\limsup\limits_{n\to\infty}\frac{1}{n}H(\bigvee\limits_{i=0}^{n-1}\varphi^{-1}(g^{-i}\alpha))$
$=\limsup\limits_{n\to\infty}\frac{1}{n}H(\bigvee\limits_{i=0}^{n-1}f^{-i}(\varphi^{-1}\alpha))=h(f,\varphi^{-1}(\alpha))$. Since φ is a homeomorphism thus $h(f)=h(g)$. □

Example 3.8. *Let $X=R[x]$ be one variable polynomial function space, $\mu:X\to[0,1]$ and $\nu:X\to[0,1]$ be defined by:*

$$\mu(f)=\begin{cases}2^{-\deg f} & if\ \deg f\neq 0\\ 0 & \text{o.w}\end{cases},$$

$$\nu(f)=\begin{cases}2^{-\frac{1}{\deg f}} & if\ \deg f\neq 0\\ 1 & \text{o.w}\end{cases},$$

also let $\mu^i:X\to[0,1]$ and $\nu^i:X\to[0,1]$ be defined by:

$$\mu^i(f)=\begin{cases}2^{-i} & \text{if deg f = i,}\\ 0 & \text{o.w}\end{cases},\quad \nu^i(f)=\begin{cases}2^{-\frac{1}{i}} & \text{if deg f = i}\\ 1 & \text{o.w}\end{cases}.$$

We have $\langle\mu^i,\nu^i\rangle\cap\langle\mu^j,\nu^j\rangle=\emptyset$ if $i\neq j$ and $\bigcup\limits_{i\in\mathbb{N}}\langle\mu^i,\nu^i\rangle=\langle\mu,\nu\rangle$.
Let $\tau_{\langle\mu,\nu\rangle}$ be a RIT topology generated by set $\{\langle\mu^i,\nu^i\rangle: i\in\mathbb{N}\}$,
$F:X\to X$ be the derivation map, i.e $F(f)=f'$, $0\le\alpha\le\frac{1}{2}$ and $\frac{1}{2}\le\beta\le 1$.
If $(\alpha,\beta)=(1,0)$, then $\langle\mu,\nu\rangle_{10}=\{f:\mu(f)>1,\nu(f)<0\}=\chi_\emptyset$.
If $(\alpha,\beta)=(0,1)$, then $\langle\mu,\nu\rangle_{01}=\{f:\mu(f)>0,\nu(f)<1\}=\{f: deg\, f\neq 0\}$.
If $0<\alpha,\beta<1$, then
$\langle\mu,\nu\rangle_{\alpha\beta}=\{f:\mu(f)>\alpha,\nu(f)<\beta\}=\{f: deg\, f<\min\{\log_2(\frac{1}{\alpha}),\frac{1}{\log_2(\frac{1}{\beta})}\}\}$.

Now we prove that $(\tau_{(\mu,\nu)})_{\alpha\beta}$ is an $(\alpha\beta,\langle\mu,\nu\rangle)$ - topology.

Let $\langle\mu^i,\nu^i\rangle_{\alpha\beta}\in(\tau_{(\mu,\nu)})_{\alpha\beta}, i\in I$. For every $i\in I$, $\langle\mu^i,\nu^i\rangle_{\alpha\beta}\subseteq\bigcup\limits_{i\in I}\langle\mu^i,\nu^i\rangle_{\alpha\beta}$ thus $\bigcup\limits_{i\in I}\langle\mu^i,\nu^i\rangle_{\alpha\beta}\subseteq\langle\bigcup\limits_{i\in I}\langle\mu^i,\nu^i\rangle\rangle_{\alpha\beta}$. Now let $deg\, f=n$ and $f\in\bigcup\limits_{i\in I}\langle\mu^i,\nu^i\rangle_{\alpha\beta}=\langle\sup\limits_{i\in I}\mu^i,\inf\limits_{i\in I}\nu^i\rangle_{\alpha\beta}$.

$$\sup_{i\in I}\mu^i(f)=\begin{cases}2^{-n} & \text{if deg f = n, n}\in\text{I}\\ 0 & \text{o.w}\end{cases},\ \inf_{i\in I}\nu^i(f)=\begin{cases}2^{-\frac{1}{n}} & \text{if deg f = n, n}\in\text{I}\\ 1 & \text{o.w}\end{cases}$$

Since $f \in \langle \underset{i\in I}{\cup} \langle \mu^i, \nu^i \rangle \rangle_{\alpha\beta}$ thus $\sup\limits_{i\in I} \mu^i(f) > \alpha$ and $\inf\limits_{i\in I} \nu^i(f) < \beta$ so $n \in I$ and $f \in \langle \mu^n, \nu^n \rangle_{\alpha\beta}$.

Now we prove that F is an $\alpha\beta$-relative continuous map.

If $degf = i+1$ and $f \in F^{-1}(\langle \mu^i, \nu^i \rangle_{\alpha\beta}) \cap \langle \mu, \nu \rangle_{\alpha\beta}$, then $\min\{2^{-(i+1)}, 2^{-i}\} > \alpha$, $\max\{2^{-\frac{1}{i+1}}, 2^{-\frac{1}{i}}\} < \beta$. This implies that $f \in \langle \mu^{i+1}, \nu^{i+1} \rangle_{\alpha\beta}$ and $F^{-1}(\langle \mu^i, \nu^i \rangle_{\alpha\beta}) \cap \langle \mu, \nu \rangle_{\alpha\beta} \subseteq \langle \mu^{i+1}, \nu^{i+1} \rangle_{\alpha\beta}$.
If $f \in \langle \mu^{i+1}, \nu^{i+1} \rangle_{\alpha\beta}$, then $degf = i+1$ and $2^{-(i+1)} > \alpha$, $2^{-\frac{1}{i+1}} < \beta$.
Therefore $f \in F^{-1}(\langle \mu^i, \nu^i \rangle_{\alpha\beta}) \cap \langle \mu, \nu \rangle_{\alpha\beta}$ and this implies that

$$\langle \mu^{i+1}, \nu^{i+1} \rangle_{\alpha\beta} \subseteq F^{-1}(\langle \mu_i, \nu_i \rangle_{\alpha\beta}) \cap \langle \mu, \nu \rangle_{\alpha\beta}.$$

So we have $\langle \mu^{i+1}, \nu^{i+1} \rangle_{\alpha\beta} = F^{-1}(\langle \mu^i, \nu^i \rangle_{\alpha\beta}) \cap \langle \mu, \nu \rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$.

Now we prove $h_{\alpha\beta}(F) = 0$.

Let $t = \min\{[\log_2(\frac{1}{\alpha})], [\frac{1}{\log_2(\frac{1}{\beta})}]\}$, and $n = \begin{cases} t-1 & \text{if t} \in \mathbb{N} \\ [t] & \text{o.w} \end{cases}$.

The finest $(\alpha\beta, \langle \mu, \nu \rangle)$ -open cover of $\langle \mu, \nu \rangle_{\alpha\beta}$ is an $(\alpha\beta, \langle \mu, \nu \rangle)$ -open cover for $\langle \mu, \nu \rangle_{\alpha\beta}$ by the form $\sigma = \{\langle \mu^1, \nu^1 \rangle_{\alpha\beta}, \langle \mu^2, \nu^2 \rangle_{\alpha\beta}, ..., \langle \mu^n, \nu^n \rangle_{\alpha\beta}\}$.

$F^{-1}(\sigma) = \{F^{-1}(\langle \mu^1, \nu^1 \rangle_{\alpha\beta}), F^{-1}(\langle \mu^2, \nu^2 \rangle_{\alpha\beta}), ..., F^{-1}(\langle \mu^n, \nu^n \rangle_{\alpha\beta})\} = \{\langle \mu^2, \nu^2 \rangle_{\alpha\beta}, ..., \langle \mu^{n+1}, \nu^{n+1} \rangle_{\alpha\beta}\}$.

Likewise, it could be proved that

$$F^{(-n-1)}(\sigma) = \{\langle \mu^{n+1}, \nu^{n+1} \rangle_{\alpha\beta}, ..., \langle \mu^{2n+1}, \nu^{2n+1} \rangle_{\alpha\beta}\}.$$

Thus $\sigma \vee F^{(-n-1)}(\sigma) = \emptyset$ and also $\sigma \vee F^{-1}(\sigma) \vee ... \vee F^{(-n-1)}(\sigma) = \emptyset$.
This implies that $H_{\alpha\beta}(\overset{n+1}{\underset{i=0}{\vee}} F^{-i}(\sigma)) = 0$. Therefore, $h_{\alpha\beta}(F, \sigma) = 0$.
Since σ is the finest $(\alpha\beta, \langle \mu, \nu \rangle)$ -open cover thus $h_{\alpha\beta}(F) = 0$.

4. **Concluding Remarks**

In this paper a mathematical model for the intuitionistic observer was proposed and based on this notion, some properties of RIT entropy with the level (α, β) and also RI entropy on RI dynamical systems with the levels (α, β) were investigated.Introducing and investigating other properties of this model could be an interesting subject for further research.

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