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MAPS PRESERVING TRIPLE PRODUCT ON RINGS

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Dedicated to sincere professor Mehdi Radjabalipour on turning 75

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Abstract. Let $\mathcal{R}$ and $\mathcal{R}'$ be two unital rings such that $\mathcal{R}$ contains a non-trivial idempotent P_1. If $\mathcal{R}$ is a prime ring, we characterize the form of bijective map $\varphi : \mathcal{R} \to \mathcal{R}'$ which satisfies $\varphi(ABP) = \varphi(A)\varphi(B)\varphi(P)$, for every $A, B \in \mathcal{R}$ and $P \in \{P_1, P_2\}$, where $P_2 := I - P_1$ and I is the unit member of $\mathcal{R}$. It is shown that φ is an isomorphism multiplied by a central element. Finally, we characterize the form of $\varphi : \mathcal{R} \to \mathcal{R}$ which satisfies $\varphi(P)\varphi(A)\varphi(P) = PAP$, for every $A \in \mathcal{R}$ and $P \in \{P_1, P_2\}$.

Keywords: Preserver problem, Ring, Triple product, Non-trivial idempotent.
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1. Introduction

Martindale in [7] shows that every multiplicative bijective map from a prime ring containing a non-trivial idempotent onto an arbitrary ring is necessarily additive. The question arises that if we change the product or the condition of the ring, "can it again be shown that it is an additive mapping?"

Many authors have studied this subject and obtained remarkable results, see [1-17] for more information. Let $\mathcal{R}$ be a ring and $A, B \in \mathcal{R}$. we denote Jordan triple product A and B by ABA. Molnár in [8] shows that every bijection between standard operator algebras preserving Jordan triple product is linear or conjugate linear. He in [9] considers that multiplicative Jordan triple isomorphisms between the sets of self-adjoint elements (respectively, the sets of positive elements) of von Neumann algebras. Also he shows that all those transformations originate from linear $*$-algebra isomorphisms and linear $*$-algebra antiisomorphisms in the case when the underlying von Neumann algebras do not have commutative direct summands. Li and Lu in [5] investigate that a bijective map ϕ between two von Neumann algebras, one of which has no central abelian projections, satisfying $\phi(A \bullet B \bullet C) = \phi(A) \bullet \phi(B) \bullet \phi(C)$ for all A, B, C in the domain, where $A \bullet B = AB + BA^*$.

Some authors consider maps that strongly preserve a certain product; for example, in [4] authors consider a surjective map ϕ on a factor von Neumann

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algebra satisfying $\phi(A)\phi(B) - \phi(B)\phi(A)^* = AB - BA^*$ for all A, B in the domain.

The purpose of this paper is characterizing the forms of maps preserving the usual triple product on rings. Also we characterize the forms of maps srongly preserving the Jordan triple product on rings not for all elements in the domain.

2. **Main Results**

Let $\mathcal{R}$ be an unital ring with unit I. Recall that an element P of $\mathcal{R}$ is an idempotent whenever $P^2 = P$ and two idempotents P_1 and P_2 of $\mathcal{R}$ are idempotents, then we will say that they are orthogonal whenever $P_1P_2 = P_2P_1 = 0$. Furthermore, two orthogonal idempotents P_1 and P_2 are called complete orthogonal idempotents if $P_1 + P_2 = I$.

Let $\mathcal{R}$ be an unital ring that contains a non-trivial idempotent P_1. Set $P_2 := I - P_1$. Denote $\mathcal{R}_{ij} := P_i\mathcal{R}P_j$, for $i, j \in \{1, 2\}$. Then $\mathcal{R} = \sum_{1\leq i,j\leq 2} \mathcal{R}_{ij}$. For every $A \in \mathcal{R}$ we can write $A = \sum_{1\leq i,j\leq 2} A_{ij}$, where $A_{ij} = P_iAP_j$ represents the elements of $\mathcal{R}_{ij}$. Denote the center of $\mathcal{R}$ by $\mathcal{Z}_{\mathcal{R}}$ which equals to $\{A \in \mathcal{R} : AB = BA,\ \forall B \in \mathcal{R}\}$. $\mathcal{R}$ is called prime if for arbitrary $A, B \in \mathcal{R}$ such that $A\mathcal{R}B = 0$, then either $A = 0$ or $B = 0$.

Throughout this chapter we assume that $\mathcal{R}$ and $\mathcal{R}^{'}$ are two unital rings. For simplicity, we denote the unit member of both rings by I.

Lemma 2.1. *Let $\mathcal{R}$ and $\mathcal{R}^{'}$ be two unital rings such that $\mathcal{R}$ contains a non-trivial idempotent P_1. Assume $\varphi : \mathcal{R} \to \mathcal{R}^{'}$ is a bijective map which satisfies*

$$\varphi(PAQ) = \varphi(P)\varphi(A)\varphi(Q)$$

for every $A \in \mathcal{R}$ and $P, Q \in \{P_1, P_2\}$, where $P_2 = I - P_1$. Then the following statements hold:

(i) $\varphi(A_{11} + A_{12} + A_{21} + A_{22}) = \varphi(A_{11}) + \varphi(A_{12}) + \varphi(A_{21}) + \varphi(A_{22})$, for each $A_{11} \in \mathcal{R}_{11}$, $A_{12} \in \mathcal{R}_{12}$, $A_{21} \in \mathcal{R}_{21}$ and $A_{22} \in \mathcal{R}_{22}$.

(ii) $\varphi(P_1)^2$ and $\varphi(P_2)^2$ are complete orthogonal idempotents.

Proof. We divide the proof into several steps.

Step 1. $\varphi(0) = 0$.

Since φ is a surjective map, there exists an element A such that $\varphi(A) = 0$. Thus

$$\varphi(PAQ) = \varphi(P)\varphi(A)\varphi(Q) = 0$$

for every $P, Q \in \{P_1, P_2\}$. Hence $\varphi(P_iAP_j) = \varphi(A) = 0$, for $i, j \in \{1, 2\}$ and then by injectivity of φ, $P_iAP_j = A$ which implies

$$A = \sum_{1\leq i,j\leq 2} A_{ij} = 4A \Rightarrow A = 0.$$

Step 2. For $B, B_1, \cdots, B_n \in \mathcal{R}$, if $\varphi(B) = \varphi(B_1) + \cdots + \varphi(B_n)$, then

$$\varphi(P_iBP_j) = \varphi(P_iB_1P_j) + \cdots + \varphi(P_iB_nP_j),$$

for all $i, j \in \{1, 2\}$.

Multiplying the equality

$$\varphi(B) = \varphi(B_1) + \cdots + \varphi(B_n),$$

by $\varphi(P_i)$ from the left side and then by $\varphi(P_j)$ from the right side implies

$$\varphi(P_i)\varphi(B)\varphi(P_j) = \varphi(P_i)\varphi(B_1)\varphi(P_j) + \cdots + \varphi(P_i)\varphi(B_n)\varphi(P_j)$$

which using assumption yields

$$\varphi(P_iBP_j) = \varphi(P_iB_1P_j) + \cdots + \varphi(P_iB_nP_j).$$

Step 3. $\varphi(A_{11}+A_{12}+A_{21}+A_{22}) = \varphi(A_{11})+\varphi(A_{12})+\varphi(A_{21})+\varphi(A_{22})$ for each $A_{11} \in \mathcal{R}_{11}$, $A_{12} \in \mathcal{R}_{12}$, $A_{21} \in \mathcal{R}_{21}$ and $A_{22} \in \mathcal{R}_{22}$. Since φ is surjective, there exists $B \in \mathcal{R}$ such that

$$\varphi(B) := \varphi(A_{11}) + \varphi(A_{12}) + \varphi(A_{21}) + \varphi(A_{22}).$$

Now by using Step 2, we have

$$\varphi(P_iBP_j) = \varphi(P_iA_{11}P_j) + \varphi(P_iA_{12}P_j) + \varphi(P_iA_{21}P_j) + \varphi(P_iA_{22}P_j),$$

for each $i, j \in \{1, 2\}$. Taking $i = j = 1$ in last equation yields

$$\varphi(B_{11}) = \varphi(A_{11}) + \varphi(0) + \varphi(0) + \varphi(0),$$

which together with injectivity of φ and Step 1, implies $B_{11} = A_{11}$. It is obtained in a similar way that $B_{12} = A_{12}$, $B_{21} = A_{21}$ and $B_{22} = A_{22}$ and these complete the proof.

Step 4. $\varphi(I)^2 = I$

Since φ is a surjective map, we can find an element $A \in \mathcal{R}$ such that $\varphi(A) = I$. We can write $A = A_{11} + A_{12} + A_{21} + A_{22}$ and by assumption

$$\begin{aligned}
\varphi(A_{11}) &= \varphi(P_1)\varphi(A)\varphi(P_1) = \varphi(P_1)^2,\\
\varphi(A_{12}) &= \varphi(P_1)\varphi(A)\varphi(P_2) = \varphi(P_1)\varphi(P_2),\\
\varphi(A_{21}) &= \varphi(P_2)\varphi(A)\varphi(P_1) = \varphi(P_2)\varphi(P_1),\\
\varphi(A_{22}) &= \varphi(P_2)\varphi(A)\varphi(P_2) = \varphi(P_2)^2.
\end{aligned}$$

By Step 3, we have

$$\varphi(A) = \varphi(A_{11}) + \varphi(A_{12}) + \varphi(A_{21}) + \varphi(A_{22})$$

and so

$$\begin{aligned}
I &= \varphi(P_1)^2 + \varphi(P_1)\varphi(P_2) + \varphi(P_2)\varphi(P_1) + \varphi(P_2)^2\\
&= (\varphi(P_1) + \varphi(P_2))^2.
\end{aligned}$$

On the other hand, by Step 3

$$\varphi(I) = \varphi(P_1 + P_2) = \varphi(P_1) + \varphi(P_2)$$

which implies $\varphi(I)^2 = I$.

Step 5. $\varphi(I) \in \mathcal{Z}_{\mathcal{R}'}$.

By assumption and Step 3, we have

$$\begin{aligned} \varphi(A) &= \varphi(A_{11}) + \varphi(A_{12}) + \varphi(A_{21}) + \varphi(A_{22}) \\ &= \varphi(P_1)\varphi(A)\varphi(P_1) + \varphi(P_1)\varphi(A)\varphi(P_2) \\ &+ \varphi(P_2)\varphi(A)\varphi(P_1) + \varphi(P_2)\varphi(A)\varphi(P_2) \\ &= [\varphi(P_1) + \varphi(P_2)]\varphi(A)[\varphi(P_1) + \varphi(P_2)] \\ &= \varphi(I)\varphi(A)\varphi(I). \end{aligned}$$

Multiplying the above equation by $\varphi(I)$ from left side and using Step 4 yields

$$\varphi(I)\varphi(A) = \varphi(A)\varphi(I).$$

This equation together with the surjectivity of φ shows that $\varphi(I) \in \mathcal{Z}_{\mathcal{R}'}$.

Step 6. $\varphi(P_1)^2$ and $\varphi(P_2)^2$ are complete orthogonal idempotents.

By assumption and Step 1 we have

$$\varphi(P_i)^4 = \varphi(P_iP_iP_i)\varphi(P_i) = \varphi(P_i)^2,$$

and

$$\varphi(P_i)^2\varphi(P_j)^2 = \varphi(P_iP_iP_j)\varphi(P_j) = \varphi(0)\varphi(P_j) = 0,$$

for $1 \leq i \neq j \leq 2$, which imply that $\varphi(P_1)^2$ and $\varphi(P_2)^2$ are orthogonal idempotents. In order to complete the proof, it is enough to show that $\varphi(P_1)^2 + \varphi(P_2)^2 = I$. It is clear that by assumption $\varphi(P_1)\varphi(P_2) = \varphi(P_1IP_2) = \varphi(0) = 0$. This together with Steps 3, 4 and 5 yields

$$\begin{aligned} I - \varphi(P_1)^2 &= I - [\varphi(I) - \varphi(P_2)]^2 \\ &= I - [I + \varphi(P_2)^2 - 2\varphi(I)\varphi(P_2)] \\ &= -\varphi(P_2)^2 + 2[\varphi(P_1) + \varphi(P_2)]\varphi(P_2) \\ &= \varphi(P_2)^2 \end{aligned}$$

and this completes the proof. □

Theorem 2.2. *Let $\mathcal{R}$ and $\mathcal{R}^{'}$ be two unital rings. Assume $\mathcal{R}$ is a prime ring and contains a non-trivial idempotent P_1 and also $\varphi : \mathcal{R} \to \mathcal{R}^{'}$ is a bijective map which satisfies*

$$\varphi(ABP) = \varphi(A)\varphi(B)\varphi(P)$$

for every $A, B \in \mathcal{R}$ and $P \in \{P_1, P_2\}$, where $P_2 := I - P_1$. Then φ is an isomorphism multiplied by a central element, that means there exists an element $R \in \mathcal{Z}_{\mathcal{R}}$ such that $R\varphi$ is an isomorphism.

Proof. We divide the proof into several steps.

Step 1. $\varphi(A_{ij} + B_{ij}) = \varphi(A_{ij}) + \varphi(B_{ij})$ for every $A_{ij}, B_{ij} \in \mathcal{R}_{ij}$, where $1 \leq i \neq j \leq 2$.

By assumption and Lemma 2.1 we have

$$\begin{aligned}
\varphi(A_{ij}+B_{ij}) &= \varphi((P_i+A_{ij})(P_j+B_{ij})P_j) \\
&= \varphi(P_i+A_{ij})\varphi(P_j+B_{ij})\varphi(P_j) \\
&= [\varphi((P_i)+\varphi(A_{ij})][\varphi(P_j)+\varphi(B_{ij})]\varphi(P_j) \\
&= \varphi(P_i)\varphi(P_j)\varphi(P_j)+\varphi(P_i)\varphi(B_{ij})\varphi(P_j) \\
&+ \varphi(A_{ij})\varphi(P_j)\varphi(P_j)+\varphi(A_{ij})\varphi(B_{ij})\varphi(P_j) \\
&= \varphi(P_iP_jP_j)+\varphi(P_iB_{ij}P_j)+\varphi(A_{ij}P_jP_j)+\varphi(A_{ij}B_{ij}P_j) \\
&= \varphi(0)+\varphi(B_{ij})+\varphi(A_{ij})+\varphi(0) \\
&= \varphi(B_{ij})+\varphi(A_{ij}).
\end{aligned}$$

Step 2. $\varphi(A_{ii}+B_{ii})=\varphi(A_{ii})+\varphi(B_{ii})$ for every $A_{ii},B_{ii}\in\mathcal{R}_{ii}$, where $i\in\{1,2\}$.

By the surjectivity of φ there exists an element T of $\mathcal{R}$ such that $\varphi(T)=\varphi(A_{ii})+\varphi(B_{ii})$. Let $C\in\mathcal{R}$. Hence

$$\begin{aligned}
\varphi(TCP_j) &= \varphi(T)\varphi(C)\varphi(P_j) \\
&= [\varphi(A_{ii})+\varphi(B_{ii})]\varphi(C)\varphi(P_j) \\
&= \varphi(A_{ii})\varphi(C)\varphi(P_j)+\varphi(B_{ii})\varphi(C)\varphi(P_j) \\
&= \varphi(A_{ii}CP_j)+\varphi(B_{ii}CP_j).
\end{aligned}$$

By Step 1 since $A_{ii}CP_j, B_{ii}CP_j\in\mathcal{R}_{ij}$, we have

$$\varphi(TCP_j)=\varphi(A_{ii}CP_j+B_{ii}CP_j),\ \text{for } j\neq i.$$

The injectivity of φ implies $(T-A_{ii}-B_{ii})CP_j=0$. Since $\mathcal{R}$ is prime, we obtain $T=A_{ii}+B_{ii}$ and this completes the proof.

Step 3. φ is additive.

Let $A,B\in\mathcal{R}$. So there exist $A_{11},B_{11}\in\mathcal{R}_{11}$, $A_{12},B_{12}\in\mathcal{R}_{12}$, $A_{21},B_{21}\in\mathcal{R}_{21}$ and $A_{22},B_{22}\in\mathcal{R}_{22}$ such that $A=\sum_{1\le i,j\le 2}A_{ij}$ and $B=\sum_{1\le i,j\le 2}B_{ij}$. By Lemma 2.1 and Steps 1 and 2 we have

$$\begin{aligned}
\varphi(A+B) &= \varphi(\sum_{1\le i,j\le 2}A_{ij}+\sum_{1\le i,j\le 2}B_{ij}) \\
&= \varphi(\sum_{1\le i,j\le 2}[A_{ij}+B_{ij}]) \\
&= \varphi(A_{11}+B_{11})+\varphi(A_{12}+B_{12})+\varphi(A_{21}+B_{21})+\varphi(A_{22}+B_{22}) \\
&= \varphi(A_{11})+\varphi(B_{11})+\varphi(A_{12})+\varphi(B_{12}) \\
&+ \varphi(A_{21})+\varphi(B_{21})+\varphi(A_{22})+\varphi(B_{22}) \\
&= \varphi(A)+\varphi(B).
\end{aligned}$$

Step 4. $\varphi(I)\varphi(AB)=[\varphi(I)\varphi(A)][\varphi(I)\varphi(B)]$ for every $A,B\in\mathcal{R}$.

By assumption we know that

$$\varphi(ABP_1)=\varphi(A)\varphi(B)\varphi(P_1),$$
$$\varphi(ABP_2)=\varphi(A)\varphi(B)\varphi(P_2).$$

The sum of these two relations together with Step 3 implies

$$\varphi(AB) = \varphi(A)\varphi(B)[\varphi(P_1) + \varphi(P_2)] = \varphi(A)\varphi(B)\varphi(I).$$

By Step 5 of the previous theorem, $\varphi(I) \in \mathcal{Z}_{\mathcal{R}'}$, so we obtain

$$\varphi(I)\varphi(AB) = [\varphi(I)\varphi(A)][\varphi(I)\varphi(B)]$$

and this completes the proof. □

Theorem 2.3. *Let $\mathcal{R}$ be an unital ring containing a non-trivial idempotent P_1 and $\varphi : \mathcal{R} \to \mathcal{R}$ be a map such that $I \in \mathrm{Im}(\varphi)$ and satisfies*

$$\varphi(P)\varphi(A)\varphi(P) = PAP$$

for every $A \in \mathcal{R}$ and $P \in \{P_1, P_2\}$, where $P_2 := I - P_1$. Then $\varphi(P_1) + \varphi(P_2)$ is invertible. Moreover, if $\varphi(P_1) \in \mathcal{Z}(\varphi(\mathcal{R}))$, then for every $A \in \mathcal{R}$

$$\varphi(A) = UAU + VAV,$$

where $U := \varphi(P_1)^2$ and $V := \varphi(P_2)^2$.

Proof. We divide the proof into several steps.
Step 1. $\varphi(P_i)\varphi(P_j) = 0$ for $i, j \in \{1, 2\}$ and $i \neq j$.
It is clear by assumption that

$$\varphi(P_i)^3 = P_i,\ i = 1, 2. \tag{1}$$

Since $I \in \mathrm{Im}(\varphi)$, there exists an element $U \in \mathcal{R}$ such that $\varphi(U) = I$. Hence

$$\varphi(Pi)^2 = \varphi(Pi)\varphi(U)\varphi(Pi) = P_iUP_i,\ i = 1, 2 \tag{2}$$

which yields

$$\varphi(Pi)^2\varphi(Pj)^2 = P_iUP_iP_jUP_j = 0,\ 1 \leq i \neq j \leq 2. \tag{3}$$

Let $i, j \in \{1, 2\}$ and $i \neq j$ and set $X := \varphi(Pj)\varphi(Pi)^2$. By (2.1), (2.3) and assumption, we have

$$P_jX = \varphi(P_j)^3\varphi(Pj)\varphi(Pi)^2 = \varphi(Pj)^4\varphi(Pi)^2 = 0,$$

$$P_iX = \varphi(P_i)^3\varphi(Pj)\varphi(Pi)^2 = \varphi(P_i)^2P_iP_jP_i\varphi(Pi) = 0$$

and then

$$X = (P_i + P_j)X = 0. \tag{4}$$

Now we show that $Y = \varphi(P_i)\varphi(P_j) = 0$. By (2.1), (2.3), (2.4) and assumption, we have

$$YPi = \varphi(P_i)\varphi(P_j)Pi = \varphi(P_i)\varphi(P_j)\varphi(Pi)^3 = P_iP_jP_i\varphi(Pi)^2 = 0,$$

$$YP_j = \varphi(P_i)\varphi(P_j)Pj = \varphi(P_i)\varphi(P_j)^4 = X\varphi(P_j)^2 = 0$$

and hence $Y = Y(P_i + P_j) = 0$.

Step 2. $\varphi(P_1)+\varphi(P_2)$ is invertible and $[\varphi(P_1)+\varphi(P_2)]^{-1}=\varphi(P_1)^2+\varphi(P_2)^2$.

By Step 1, it is clear that

$$[\varphi(P_1)+\varphi(P_2)]^3=\varphi(P_1)^3+\varphi(P_2)^3=P_1+P_2=I \tag{5}$$

which implies that $\varphi(P_1)+\varphi(P_2)$ is invertible and $[\varphi(P_1)+\varphi(P_2)]^{-1}=[\varphi(P_1)+\varphi(P_2)]^2$ and again by using Step 1, it is equal to $\varphi(P_1)^2+\varphi(P_2)^2$.

Step 3. $\varphi(I)=\varphi(P_1)+\varphi(P_2)$.

By assumptions, we have

$$[\varphi(P_1)+\varphi(P_2)]\varphi(I)[\varphi(P_1)+\varphi(P_2)]=P_1+P_2$$
$$+\varphi(P_1)\varphi(I)\varphi(P_2)+\varphi(P_2)\varphi(I)\varphi(P_1)$$
$$=P_1+P_2+\varphi(I)\varphi(P_1)\varphi(P_2)+\varphi(P_2)\varphi(P_1)\varphi(I)$$
$$=P_1+P_2=I.$$

Multiplying by $[\varphi(P_1)+\varphi(P_2)]^2$ from two sides together with (2.5) and Step 1 yields

$$\varphi(I)=[\varphi(P_1)+\varphi(P_2)]^4=\varphi(P_1)+\varphi(P_2).$$

Step 4. If $\varphi(P_1)\in\mathcal{Z}(\varphi(\mathcal{R}))$, then for every $A\in\mathcal{R}$

$$\varphi(A)=UAU+VAV,$$

where $U=\varphi(P_1)^2$ and $V=\varphi(P_2)^2$.

Let $A\in\mathcal{R}$. Assumptions and Steps 1 and 3 imply

$$\varphi(I)\varphi(A)\varphi(I)=[\varphi(P_1)+\varphi(P_2)]\varphi(A)[\varphi(P_1)+\varphi(P_2)]$$
$$=P_1AP_1+P_2AP_2+\varphi(P_1)\varphi(A)\varphi(P_2)+\varphi(P_2)\varphi(A)\varphi(P_1)$$
$$=P_1AP_1+P_2AP_2+\varphi(A)\varphi(P_1)\varphi(P_2)+\varphi(P_2)\varphi(P_1)\varphi(A)$$
$$=P_1AP_1+P_2AP_2.$$

By (2.2), $P_i\varphi(P_i)^2=\varphi(P_i)^2P_i=P_iUP_i=\varphi(P_i)^2$, for $i\in\{1,2\}$. This together with (2.1) and multiplying the last equation by $\varphi(I)^{-1}=\varphi(P_1)^2+\varphi(P_2)^2$ implies

$$\varphi(A)=\varphi(P_1)^2P_1AP_1\varphi(P_1)^2+\varphi(P_2)^2P_1AP_1\varphi(P_2)^2$$
$$+\varphi(P_1)^2P_2AP_2\varphi(P_1)^2+\varphi(P_2)^2P_2AP_2\varphi(P_2)^2$$
$$=\varphi(P_1)^2A\varphi(P_1)^2+\varphi(P_2)^2A\varphi(P_2)^2$$

and this completes the proof. □

3. Aknowledgement

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