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ON APPROXIMATE ORTHOGONALLY RING HOMOMORPHISMS AND ORTHOGONALLY RING DERIVATIONS

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Dedicated to sincere professor Mehdi Radjabalipour on turning 75

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Abstract. Using fixed point methods, we prove the stability of orthogonally ring homomorphism and orthogonally ring derivation in Banach algebras.

Keywords: Stability; Banach algebras; Fixed point approach; Ring derivations; Ring homomorphisms.
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1. Introduction

The store of stability of functional equation dates back to 1925, when a stability result appeared in the celebrated book Polya and Szego [28]. In 1940, Ulam [36] posed the famous Ulam stability problem which was partially solved by Heyrs [23] in the framework of Banach spaces. For more details about the result concerning such problems, we refer the reader to ([2], [4], [18], [19], [30]-[35]).

Let A, B be two algebras. A mapping $f : A \to B$ is called ring homomorphism if f is an additive mapping satisfying $f(xy) = f(x)f(y)$ for all $x, y \in A$. An additive mapping $f : A \to A$ is called a ring derivation if $f(xy) = xf(y) + f(x)y$ holds for all $x, y \in A$. If, in addition, $f(\lambda x) = \lambda f(x)$ for all $x \in A$ and all $\lambda \in \mathbb{F}$, then f is called a linear derivation, where $\mathbb{F}$ denotes the scalar field of A. The various problems of the stability of derivations have been studied during last few years (see for instance, [3], [14], [17], [24], [27]).

In 2003, Radu [29], employed the alternative fixed point theorem, due to Diaz and Margolis [25], to prove the stability of the Cauchy additive function equation. Subsequently, this method was applied to investigate the Hyers-Ulam stability for the Jensen functional equation [9], as well as for the Cauchy functional equation [8], by considering a general control function $\varphi(x, y)$ with suitable

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properties.Using such an elegant idea, several authors applied the method to investigate the stability of some functional equations (see [7], [10], [16], [20], [26]).

Recently Eshaghi and et al. introduced orthogonally sets and proved the real generalization of Banach fixed point theorem on this sets [15], we review the basic definitions of orthogonally sets, which can be consider the main definition of our paper (See [1], [6], [11]- [13], [21], [22]).

Definition 1.1. Let $X \neq \emptyset$ and $\perp \subseteq X \times X$ be an binary relation. If $\perp$ satisfies the following condition

$$\exists x_0; (\forall y; y \perp x_0) \ \ or \ \ (\forall y; x_0 \perp y),$$

it is called an orthogonally set (briefly O-set). We denote this O-set by $(X, \perp)$.

Definition 1.2. Let $(X, \perp)$ be an O-set. A sequence $\{x_n\}_{n\in\mathbb{N}}$ is called orthogonally sequence (briefly O-sequence) if

$$(\forall n; x_n \perp x_{n+1}) \ \ or \ \ (\forall n; x_{n+1} \perp x_n).$$

Definition 1.3. Let $(X, \perp, d)$ be an orthogonally metric space ($(X, \perp)$ is an O-set and (X, d) is a metric space). Then $f : X \to X$ is $\perp-$continuous in $a \in X$ if for each O-sequence $\{a_n\}_{n\in\mathbb{N}}$ in X if $a_n \to a$, then $f(a_n) \to f(a)$. Also f is $\perp-$continuous on X if f is $\perp-$continuous on each $a \in X$.

It is easy to see that every continuous mapping is $\perp-$continuous.

Definition 1.4. Let $(X, \perp, d)$ be an orthogonally metric space, then X is orthogonally complete (briefly O-complete) if every Cauchy O-sequence is convergent.

It is easy to see that every complete metric space is O-complete and the converse is not true.

Definition 1.5. Let $(X, \perp, d)$ be an orthogonally metric space and $0 < \lambda < 1$. A mapping $f : X \to X$ is said to be orthogonality contraction with Lipschitz constant λ if

$$d(fx, fy) \leq \lambda d(x, y) \quad if \ x \perp y.$$

Let H be a Hilbert space. Suppose that $f : H \to \mathbb{C}$ is a mapping satisfying

$$f(x) = \|x\|^2 \tag{1}$$

for all $x \in X$. It is natural that this equation is a quadratic functional equation. On the other hand by considering $x \perp y$ with $< x, y >= 0$ for $x, y \in H$, it is easy to see that the above function $f : H \to \mathbb{C}$ is an orthogonally additive functional equation, that is $f(x+y) = f(x)+f(y)$ if $x \perp y$. This means that orthogonality may change a functional equation.

Recently, Bahraini and et al. [5], proved a fixed point theorem in O-sets as follows:

Theorem 1.6. *Let $(X, d, \perp)$ be an O-complete generalized metric space. Let $T : X \to X$ be a $\perp$-preserving, $\perp$-continuous and $\perp$-λ-contraction. Let $x_0 \in X$ satisfies for all $y \in X$, $x_0 \perp y$ or for all $y \in X$, $y \perp x_0$, and consider the "O-sequence of successive approximations with initial element x_0 ": x_0, $T(x_0)$, $T^2(x_0)$, ..., $T^n(x_0)$, ... Then, either $d(T^n(x_0), T^{n+1}(x_0)) = \infty$ for all $n \geq 0$, or there exists a positive integer n_0 such that $d(T^n(x_0), T^{n+1}(x_0)) < \infty$ for all $n > n_0$. If the second alternative holds, then*
i): the O-sequence of $\{T^n(x_0)\}$ is convergent to a fixed point x^* of T.
ii): x^* is the unique fixed point of T in $X^* = \{y \in X : d(T^n(x_0), y) < \infty\}$.
iii): If $y \in X$, then

$$d(y, x^*) \leq \frac{1}{1-\lambda} d(y, T(y)).$$

Example 1.7. *Let $X = [0, 1)$ and let the metric on X be the euclidian metric. Define $x \perp y$ if $xy \in \{x, y\}$ for all $x, y \in X$. Let $f : X \to X$ be a mapping defined by*

$$f(x) = \begin{cases} \frac{x}{2} \; if \; x \in \mathbb{Q} \cap X \\ 0 \; if \; x \in \mathbb{Q}^c \cap X. \end{cases}$$

It is easy to see that X is O-complete (not complete), f is $\perp$-continuous (not continuous on X), $\perp$-λ-contraction for $\lambda = \frac{1}{2}$ (not contraction on X) and $\perp$-preserving on X. By our theorem, f has a unique fixed point. However, f is not a contraction on X and so by Theorem Diaz and Margolis we cannot find any fixed point for f.

In this paper, we prove the stability of orthogonally ring homomorphism and orthogonally ring derivation in Banach algebras.

2. Main results

In this section, we suppose that $(X, \|.\|_1, \perp_1)$ is an orthogonality Banach algebra with $\perp_1 := \acute{\perp}_1 \bigcup \{(x, x) : x \in X\}$. Also, we suppose that $(Y, \|.\|_2, \perp_2)$ is an orthogonality Banach algebra with $\perp_2 := \acute{\perp}_2 \bigcup \{(y, y) : y \in Y\}$.

Theorem 2.1. *Suppose that $f : X \to Y$ is a mapping for which there exists a function $\varphi : X^4 \to [0, \infty)$ such that*

$$\|f(x + y) + f(ab) - f(x) - f(y) - f(a)f(b)\|_2 \leq \varphi(x, y, a, b), \tag{2}$$

for all $x, y, a, b \in X$ with $x \perp_1 y$ and $a \perp_1 b$. Suppose that there exists $L < 1$ such that

$$\varphi(x, y, a, b) \leq 2L\varphi(\frac{x}{2}, \frac{y}{2}, \frac{a}{2}, \frac{b}{2}) \tag{3}$$

for all $x, y, a, b \in X$ with $x \perp_1 y$ and $a \perp_1 b$. Then there exists a unique orthogonally ring homomorphism mapping $H : X \to Y$ such that

$$\|H(x) - f(x)\|_2 \leq \frac{1}{1-L}\varphi(x, x, 0, 0) \tag{4}$$

for all $x \in X$.

Proof. Putting $a = b = 0$ and $x = y$ in ((2)) by definition $\perp_1$, we get

$$\|f(2x) - 2f(x)\|_2 \leq \varphi(x, x, 0, 0)$$

for all $x \in X$. Hence we have

$$\|\frac{f(2x)}{2} - f(x)\|_2 \leq \frac{1}{2}\varphi(x, x, 0, 0) < \varphi(x, x, 0, 0) \tag{5}$$

for all $x \in X$. Consider the set

$$\Omega := \{g : \ \ g : X \to Y, \ \ g(x) \perp_2 \frac{1}{2}g(2x) \ \ or \ \ \frac{1}{2}g(2x) \perp_2 g(x), \ \ \forall x \in X\}.$$

For every $g, h \in \Omega$, define

$$d(g, h) = inf\{K \in (0, \infty) : \ \ \|g(x) - h(x)\|_2 \leq K\varphi(x, x, 0, 0), \ \ \forall x \in X\}$$

Now, we put the $\perp$ relation orthogonal continuous on Ω as follows: for all $g, h \in \Omega$

$$h \perp g \ \ \Leftrightarrow \ \ (h(x) \perp_2 g(x) \ \ or \ \ g(x) \perp_2 h(x) \ \ \forall x \in X)$$

We show that $(\Omega, d, \perp)$ is an O-complete generalized metric space.Let$\{f_n\}_{n\in N}$ be an O-cauchy sequence in $(\Omega, d, \perp)$.Then for each $\epsilon > 0$,there is some $n_0 \in N$ such that

$$d(f_n, f_m) < \epsilon \qquad\qquad m, n \geq n_0$$

By the definition , for each $m, n \geq n_0$,

$$\|f_n(x) - f_m(x)\|_2 \leq \epsilon\varphi(x, x, 0, 0) \qquad\qquad x \in X \tag{6}$$

Hence, for each $x \in X$, $\{f_n(x)\}_{n\in N}$ is an O-cauchy sequence in cmplete metric space Y. It follows that $f(x) = \lim_{n\to\infty} f_n(x)$ exists for all $x \in X$. By ((6)) for each $n \geq n_0$

$$\|f(x) - f_n(x)\|_2 = \lim_{m\to\infty} \|f_m(x) - f_n(x)\|_2 \leq \epsilon\varphi(x, x, 0, 0) \qquad\qquad x \in X$$

Therefore $d(f, f_n) < \epsilon$ for each $n \geq n_0$. Hence $(\Omega, d, \perp)$ is an O-complete generalized metric space.

Now, we consider the $J : \Omega \to \Omega$ such that

$$J(g)(x) := \frac{1}{2}g(2x)$$

for all $x \in X$. For any $g, h \in \Omega$ with $g \perp h$, it follows that for all $x \in X$

$$\begin{aligned} d(g,h) < K &\Rightarrow \|g(x) - h(x)\|_2 \leq K\varphi(x,x,0,0) \\ &\Rightarrow \|\frac{g(2x)}{2} - \frac{h(2x)}{2}\|_2 \leq K\frac{\varphi(2x,2x,0,0)}{2} \\ &\Rightarrow \|Jg(x) - Jh(x)\|_2 \leq LK\varphi(x,x,0,0). \end{aligned}$$

Hence we have

$$d(J(g), J(h)) \leq Ld(g,h).$$

It is to show that J is a $\perp$-preserving mapping. Now, we show that J is a $\perp$-continuous function. To this end, let $\{g_n\}_{n\in N}$ be a O-sequence with $g_n \perp g_{n+1}$ or $g_{n+1} \perp g_n$ in $(\Omega, d, \perp)$ which convergent to $g \in \Omega$ and let $\epsilon > 0$ be given. Then there exists $N \in \mathbb{N}$ and $K \in R^+$ with $K < \epsilon$ such that

$$\|g_n(x) - g(x)\|_2 \leq K\varphi(x,x,0,0)$$

for all $x \in X$ and $n \geq N$, and so

$$\|\frac{g_n(2x)}{2} - \frac{g(2x)}{2}\|_2 \leq K\frac{\varphi(2x,2x,0,0)}{2}$$

for all $x \in X$ and $n \geq N$. By inequality ((3)) and the define of J, we get

$$\|J(g_n)(x) - J(g)(x)\|_2 \leq LK\varphi(x,x,0,0)$$

for all $x \in X$ and $n \geq N$. Hence

$$d(J(g_n), J(g)) \leq LK < \epsilon$$

for all $n \geq N$. It follows that J is $\perp$-continuous. By definition Ω, we have $f \perp_2 J(f)$ or $J(f) \perp_2 f$, by applying the inequality ((5)), we see that $d(J(f), f) \leq 1$. It follows from Theorem ((1.6)), that J has a unique fixed point $H : X \to Y$ in the set $\Lambda = \{g \in \Omega : d(g,f) < \infty\}$, where H is defined by

$$H(x) := \lim_{n\to\infty} \frac{f(2^n x)}{2^n} \tag{7}$$

for all $x \in X$. By Theorem ((1.6)),

$$d(H,f) \leq \frac{1}{1-L}.$$

It follow from ((3)) that

$$\lim_{n\to\infty} \frac{1}{2^n}\varphi(2^n x, 2^n y, 2^n a, 2^n b) = 0. \tag{8}$$

It follow from ((2)), ((7)) and ((8)) that

$$\begin{aligned} \|H(x+y) - H(x) - H(y)\|_2 &= \lim_{n\to\infty} \frac{1}{2^n}\|f(2^n(x+y)) - f(2^n x) - f(2^n y)\|_2 \\ &\leq \lim_{n\to\infty} \frac{1}{2^n}\varphi(2^n x, 2^n y, 0, 0) = 0. \end{aligned}$$

for all $x, y \in X$ with $x \perp y$. Also, we get

$$\begin{aligned}\|H(ab) - H(a)H(b)\|_2 &= \lim_{n\to\infty} \frac{1}{4^n}\|f(4^n(ab)) - f(2^n a)f(2^n b)\|_2 \\ &\leq \lim_{n\to\infty} \frac{1}{2^n}\varphi(0, 0, 2^n a, 2^n b) = 0\end{aligned}$$

for all $a, b \in X$ with $a \perp b$. This show that H is orthogonally ring homomorphism. □

Corollary 2.2. *Let $\epsilon \in [0, \infty)$ and $f : X \to Y$ be a mapping such that*

$$\|f(x + y) + f(ab) - f(x) - f(y) - f(a)f(b)\|_2 \leq \epsilon$$

for all $x, y, a, b \in X$ with $x\perp_1 y$ and $a\perp_1 b$. Then there exists a unique orthogonally ring homomorphism mapping $H : X \to Y$ such that

$$\|H(x) - f(x)\|_2 \leq \epsilon$$

for all $x \in X$.

Proof. Set $\varphi(x, y, a, b) = \frac{\epsilon}{2}$ for all $x, y, a, b \in X$ with $x\perp_1 y$ and $a\perp_1 b$ and let $L = \frac{1}{2}$ in Theorem ((2.1)). Then we get the desired result. □

Corollary 2.3. *Let $p \in (0, 1)$ and $\epsilon \in [0, \infty)$. Suppose that $f : X \to Y$ is a mapping such that*

$$\|f(x + y) + f(ab) - f(x) - f(y) - f(a)f(b)\|_2 \leq \epsilon(\|x\|_1^p + \|y\|_1^p + \|a\|_1^p + \|b\|_1^p)$$

for all $x, y, a, b \in X$ with $x\perp_1 y$ and $a\perp_1 b$. Then there exists a unique orthogonally ring homomorphism mapping $H : X \to Y$ such that

$$\|H(x) - f(x)\|_2 \leq \frac{2\epsilon}{2 - 2^p}\|x\|_1^p$$

for all $x \in X$.

Proof. Set $\varphi(x, y, a, b) = \epsilon(\|x\|_1^p + \|y\|_1^p + \|a\|_1^p + \|b\|_1^p)$ for all $x, y, a, b \in X$ with $x\perp_1 y$ and $a\perp_1 b$ and let $L = 2^{p-1}$ in Theorem ((2.1)). Then we get the desired result. □

Theorem 2.4. *Suppose that $f : X \to X$ is a mapping for which there exists a function $\varphi : X^4 \to [0, \infty)$ such that*

$$\|f(x + y) + f(ab) - f(x) - f(y) - bf(a) - af(b)\|_1 \leq \varphi(x, y, a, b), \tag{9}$$

for all $x, y, a, b \in X$ with $x\perp_1 y$ and $a\perp_1 b$. Suppose that there exists $L < 1$ such that

$$\varphi(x, y, a, b) \leq 2L\varphi(\frac{x}{2}, \frac{y}{2}, \frac{a}{2}, \frac{b}{2}) \tag{10}$$

for all $x, y, a, b \in X$ *with* $x \perp_1 y$ *and* $a \perp_1 b$*. Then there exists a unique orthogonally ring derivation mapping* $D : X \to X$ *such that*

$$\|D(x) - f(x)\|_1 \le \frac{1}{1-L}\varphi(x, x, 0, 0) \tag{11}$$

for all $x \in X$.

Proof. By the reasoning as that in the proof Theorem ((2.1)), there exists a unique orthogonally ring derivation mapping $D : X \to X$ satisfying ((11)). The mapping $D : X \to X$ is given by $D(x) := \lim_{n\to\infty} 2^{-n} f(2^n x)$ for all $x \in X$. It follows from ((9)),

$$\begin{aligned}\|D(ab) - bD(a) - aD(b)\|_1 &= \lim_{n\to\infty} \frac{1}{4^n}\|f(4^n(ab)) - 2^n bf(2^n a) - 2^n af(2^n b)\|_1 \\ &\le \lim_{n\to\infty} \frac{1}{2^n}\varphi(0, 0, 2^n a, 2^n b) = 0\end{aligned}$$

for all $a, b \in X$ with $a \perp b$. This show that H is orthogonally ring derivation. □

Corollary 2.5. *Let* $\epsilon \in [0, \infty)$ *and* $f : X \to X$ *be a mapping such that*

$$\|f(x + y) + f(ab) - f(x) - f(y) - bf(a) - af(b)\|_1 \le \epsilon$$

for all $x, y, a, b \in X$ *with* $x \perp_1 y$ *and* $a \perp_1 b$*. Then there exists a unique orthogonally ring derivation mapping* $D : X \to X$ *such that*

$$\|D(x) - f(x)\|_1 \le \epsilon$$

for all $x \in 1$.

Proof. Set $\varphi(x, y, a, b) = \frac{\epsilon}{2}$ for all $x, y, a, b \in X$ and let $L = \frac{1}{2}$ in Theorem ((2.4)). Then we get the desired result. □

Corollary 2.6. *Let* $p \in (0, 1)$ *and* $\epsilon \in [0, \infty)$*. Suppose that* $f : X \to X$ *is a mapping such that*

$$\|f(x+y)+f(ab)-f(x)-f(y)-bf(a)-af(b)\|_1 \le \epsilon(\|x\|_1^p+\|y\|_1^p+\|a\|_1^p+\|b\|_1^p)$$

for all $x, y, a, b \in X$ *with* $x \perp_1 y$ *and* $a \perp_1 b$*. Then there exists a unique orthogonally ring derivation mapping* $D : X \to X$ *such that*

$$\|D(x) - f(x)\|_1 \le \frac{2\epsilon}{2 - 2^p}\|x\|_1^p$$

for all $x \in X$.

Proof. Set $\varphi(x, y, a, b) = \epsilon(\|x\|_1^p + \|y\|_1^p + \|a\|_1^p + \|b\|_1^p)$ for all $x, y, a, b \in X$ and let $L = 2^{p-1}$ in Theorem (2.4). Then we get the desired result. □

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References

[1] N. Ansari, M.H. Hooshmand, M. Eshaghi Gordji, K. Jahedi, *Stability of fuzzy orthogonally -n-derivation in orthogonally fuzzy C*-algebras*, J. Nonlinear Anal. Appl. 12 (2021) 533–540 .

[2] T. Aoki, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan 2 (1950) 64–66.

[3] R. Badora, *On approximate derivations*, Math. Inequal. Appl. 9 (2006) 167–173.

[4] A. Bahraini, G. Askari, *Homogeneous Equation Between Banach -Modules*, Journal of Mathematical Analysis 9 (2018) 11–18.

[5] A. Bahraini, G. Askari, M. Eshaghi Gordji, R. Gholami, *Stability and hyperstability of orthogonally ∗-m-homomorphisms in orthogonally Lie C^*-algebras: a fixed point approach* , J. Fixed Point Theory Appl. (2018), 1–12.

[6] H. Baghani, M. E. Gordji, M. Ramezani, *Orthogonal sets: The axiom of choice and proof of a fixed point theorem*, J. Fixed Point Theory Appl., 18 (2016), 465–477.

[7] M. Bavand Savadkouhi, M.E. Gordji, J.M. Rassias, N. Ghobadipour, *Approximate ternary Jordan derivations on Banach ternary algebras*, J. Math. Phys. 50 (2009), 20–29.

[8] L. Cădariu and V. Radu, *On the stability of the Cauchy functional equation: A fixed point approach*, Grazer Math. Ber. 346 (2004).

[9] L. Cădariu, V. Radu, *Fixed points and the stability of Jensens functional equation*, J. Ineq. Pure Appl. Math. 4 (2003).

[10] L. Cădariu, V. Radu, *Fixed Point Methods for the Generalized Stability of Functional Equations in a Single Variabl*, Fixed Point Theory Appl. (2008) 1–15.

[11] M. Eshaghi, H. Habibi, *Existence and uniqueness of solutions to a first-order differential equation via fixed point theorem in orthogonal metric space*, FACTA UNIVERSITATIS (NIS) Ser. Math. 34 (2019) 123-135.

[12] M. Eshaghi Gordji, H Habibi, *Fixed point theory in generalized orthogonal metric space*, J. Linear Topol. Algeb., 6 (2017), 251–260.

[13] M. Eshaghi Gordji, H. Habibi and M.B. Sahabi, *Orthogonal sets; orthogonal contractions*, Asian-Eur. J. Math. 12 (2019).

[14] M. Eshaghi Gordji, G. Askari, N. Ansari, G. A. Anastassiou and C. Park, *Stability and hyperstability of generalized orthogonally quadratic ternary homomorphisms in non-Archimedean ternary Banach algebras: a fixed point approach*, J. Computational Analysis And Applications, 21 (2016) 1–8.

[15] M. Eshaghi Gordji, M. Ramezani, M. De La Sen, and Y.J. Cho, *On orthogonal sets and Banach fixed point theorem*, Fixed Point Theory, 18 (2017), 569–578.

[16] M. Eshaghi Gordji, R. Farokhzad and S. A. R. Hosseinioun, *Ternary (σ, τ, ξ)-derivations on Banach ternary algebras*, Int. J. Nonlinear Anal. Appl., 5 (2014) 23–35.

[17] R. Farokhzad, S.A.R. Hosseinioun, *Perturbations of Jordan higher derivations in Banach ternary algebras: An alternative fixed point approach*, Int. J. Nonlinear Anal. Appl. 1 (2010) 42–53.

[18] P. Găvruta, *An answer to a question of John M. Rassias concerning the stability of Cauchy equation, In Advances in Equation and Inequalities*, Hardronic Math Sel., (1999) 67–71.

[19] P. Găvruta, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. 184 (1994) 431–436.

[20] P. Găvruta, L. Găvruta, *A new method for the generalized Hyers-Ulam-Rassias stability*, Int. J. Nonlinear Anal. Appl. 1 (2010) 11–18.

[21] R. Gholami, G. Askari, M. Eshaghi Gordji, *Stability and hyperstability of orthogonally ring -n-derivations and orthogonally ring -n-homomorphisms on C* algebras*, J. Linear Topol. Algebra, 7 (2018), 109–119.

[22] H. Hosseini, M. Eshaghi, *Fixed Point Results in Orthogonal Modular Metric Spaces*, J. Nonlinear Anal. Appl. 11 (2020) 425–436.
[23] D.H. Hyers, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. 27 (1941) 222–224.
[24] Y.S. Jung, *On the generalized Hyers-Ulam stability of module left derivations*, J. Math. Anal. Appl. 339 (2008) 108-114.
[25] B. Margolis, J.B. Diaz, *A fixed point theorem of the alternative for contractions on the generalized complete metric space*, Bull. Amer. Math. Soc. 126 (1968), 305–309.
[26] T. Miura, G. Hirasawa, S.-E. Takahasi, *A perturbation of ring derivations on Banach algebras*, J. Math. Anal. Appl. 319 (2006) 522–530.
[27] C. Park, M. Eshaghi Gordji, Comment on "Approximate ternary Jordan derivations on Banach ternary algebras Bavand Savadkouhi et al. J. Math. Phys. 50, (2009)] J. Math. Phys. 51 (2010).
[28] G. Polya, G. Szego, *Aufgaben und lehrsatze der Analysis*, Springer, Berlin, 1925.
[29] V. Radu, *The fixed point alternative and the stability of functional equations*, Sem. Fixed Point Theory 4 (2003) 91–96.
[30] J.M. Rassias, *Complete solution of the multi-dimensional of Ulam*, Discuss. Math. 14 (1994) 101–107.
[31] J.M. Rassias, *On Approximation of approximately linear mappings by linear mappings*, J. Funct. Anal. 46 (1982) 126–130.
[32] J.M. Rassias, *On stability of the Euler-Lagrange functional equation*, Chin. J. Math 20 (1992) 185–190.
[33] Th.M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., 72 (1978) 297–300.
[34] J.M. Rassias, *Solution of a problem of Ulam*, J. Approx. Theory 57 (1989) 268–273.
[35] P. Šemrl, *The functional equation of multiplicative derivation is superstable on standard operator algebras*, Integral Equations and Operator Theory 18 (1994) 118–122.
[36] S.M. Ulam, *Problems in Modern Mathematics*, Wiley, New York, 1964.

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