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HAUSDORFF SPACE BASED ON KM-SINGLE VALUED NEUTROSOPHIC SPACE

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Abstract. This paper introduces a novel concept of KM-single valued neutrosophic Hausdorff space and KM-single valued neutrosophic manifold space. This study generalizes the concept of KM-single valued neutrosophic manifold space to union and product of KM-single valued neutrosophic manifold space and in this regard investigates some product of KM-single valued neutrosophic manifold spaces. Indeed, this study analyses the notation of KM-single valued neutrosophic manifold based on a valued-level subset.

Keywords: KM-single valued neutrosophic space, triangular norm, KM-single valued neutrosophic Hausdorff space, KM-single valued neutrosophic manifold space.
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1. Introduction

As a generalization of the classical set theory, fuzzy set theory was introduced by Zadeh to deal with uncertainties [16]. Fuzzy set theory is playing an important role in modeling and controlling unsure systems in nature, society, and industry. The fuzzy set theory also plays a vital role in complex phenomena which is not easily characterized by classical set theory. After the pioneering work of Zadeh, there has been a great effort to obtain fuzzy analogs of classical theories. Among other fields, progressive developments are made in the field of fuzzy topology. Fuzzy topology is a fundamental branch of fuzzy theory that has become an area of active research in the last area because of its wide range of applications. One of the most important problems in fuzzy topology is to obtain a proprieties concept of fuzzy metric space. This problem has been investigated by many authors from different viewpoints . In particular, George and Veeramani have introduced and studied a notion of fuzzy metric space concerning the concept of t-norms. Furthermore, the class of topological spaces that are fuzzily metrizable agrees with the class of metrizable- topological spaces. This result permits Gregori and Romaguera to restate some classical theorems on metric completeness and metric (pre) compactness in the

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realm of fuzzy metric spaces [2]. Kaleva generalized the notion of the metric space by setting the distance between two points to be a non-negative fuzzy number. by defining an ordering and an addition in the set of fuzzy numbers, they obtained a triangle inequality which is analogous to the ordinary triangle inequality [6]. Historically, the notion of a differentiable manifold, that is, a set that looks locally like Euclidean Space, has been an integral part of various fields of mathematics. One may note their applications in the fields of differential topology [4], algebraic geometry, algebraic topology, and lie groups and their associated algebras. They will base our work upon the already well-established fuzzy structures, fuzzy topological spaces, fuzzy topological vector spaces, fuzzy derivatives. However, the definition of a fuzzy derivative provided in Foster [4], is not easily generalized to general k derivatives. Consequently, the existence of a fuzzy differentiable manifold of class greater than one has not yet been established. They shall give topological separation axioms that have not been given previously, for sake of completeness. Our principal approach is quite similar to the methods used in [4]. Namely, they shall take definitions in [1] of fuzzy continuity and fuzzy topological vector space and use these notions to give a fuzzy topological vector space differential structure by constructing a fuzzy homeomorphism and, naturally, a fuzzy diffeomorphism of class k. To do so, they provide a new definition of a fuzzy object, known as fuzzy vectors. They then define proper fuzzy directional derivatives along these abstract fuzzy vectors, and allude to their applications in manifold learning. For completeness, they shall define tangent vector spaces to these fuzzy manifolds. Further materials regarding fuzzy topological spaces are available in the literature too [5–12, 14, 15].

Regarding these points, we introduce the concept of KM-single valued neutrosophic Hausdorff space and KM-single valued neutrosophic manifold space. One of our motivations for this work is the construction of finite KM-single valued neutrosophic metric space. This study presents a concept of KM-single valued neutrosophic Hausdorff space and KM-single valued neutrosophic manifold space as a generalization of fuzzy Hausdorff space and fuzzy manifold space.

Motivation The main motivation of this work is a connection between fuzzy subsets, single-valued neutrosophic subsets, KM-single valued neutrosophic and geometry structures, specially Hausdorff space and fuzzy manifold space and KM-single valued neutrosophic manifold space. We belived that valued-cuts have a main role in construction of topological space and Hausdorff space, so with respect to the large extension of fuzzy subsets, single-valued neutrosophic subsets, KM-single valued neutrosophic we motivated to make a relation between of fuzzy subsets, single-valued neutrosophic subsets, KM-single valued neutrosophic and Hausdorff space.

2. **Preliminaries**

In this section, we recall some definitions and results, which we need in what follows [3, 13].

Theorem 2.1. *[3](Inverse Function Theorem) Let U be an open subset of $\mathbb{R}^n$, and $p \in U$. Let $g : U \longrightarrow \mathbb{R}^n$ be a smooth map. If $dg_p \in Hom(\mathbb{R}^n, \mathbb{R}^n)$ is a linear isomorphism, then there exist open neighbourhoods U_0, V_0 of $p, g(p)$ such that $g|_{U_0} : U_0 \longrightarrow V_0$ is a diffeomorphism.*

Definition 2.2. [3]Let M be a Hausdorff topological space. We say that M is an n-dimensional topological manifold if it satisfies the following condition: for any $p \in M$, there exists
(1) an open subset U with $p \in U \subseteq M$,
(2) an open subset $E \subseteq \mathbb{R}^n$, and
(3) a homeomorphism $\psi : U \longrightarrow E$.
Such a U is called a (local) coordinate neighbourhood, and ψ is called a (local) coordinate function. We write $x = \psi(p)$ and regard $(x_1, \dots, x_n)$ as local coordinates for the manifold M.

Definition 2.3. [3]Let M be a topological manifold. Let A be a set. We say that S is a C^0-atlas (or coordinate neighbourhood system) for M if $S = \{(U_\alpha, \psi_\alpha) | \alpha \in A\}$ where
(1) U_α is an open subset of M, for all $\alpha \in A$,
(2) $\psi_\alpha : U_\alpha \to E_\alpha$ is a homeomorphism to an open subset E_α of $\mathbb{R}^n$, for all $\alpha \in A$,
(3) $\bigcup_{\alpha \in A} U_\alpha = M$.

Definition 2.4. [3]Let S be a C^0-atlas for M. If $\psi_\alpha \circ \psi_\beta^{-1}$ is a C^∞-map for all $\alpha, \beta \in A$, we say that M is a C^∞-atlas for M. We call $\psi_\alpha \circ \psi_\beta^{-1}$ a coordinate transformation or transition function. The domain of the map $\psi_\alpha \circ \psi_\beta^{-1}$ is assumed to be $\psi_\beta(U_\alpha \cap U_\beta)$ (which could be the empty set). Thus, $\psi_\alpha \circ \psi_\beta^{-1}$ is a homeomorphism from $\psi_\beta(U_\alpha \cap U_\beta)$ to $\psi_\alpha(U_\alpha \cap U_\beta)$.

Definition 2.5. [3]Let M be a topological manifold. Let S be a C^∞-atlas for M. We say that M is a C^∞-manifold (or smooth manifold, or differentiable manifold). The concept of C^r-manifold can be defined in a similar way. However, from now on, manifold will always mean C^∞-manifold. The concept of complex manifold can be defined in a similar way, using coordinate charts $\psi : U \to \mathbb{C}^n$. However, the term complex manifold will always mean complex manifold with holomorphic (complex analytic) transition functions.

Definition 2.6. [3]Let M, M' be smooth manifolds with $\dim M = n, \dim M' = n'$. Let $\phi : M \to M'$ be a map. Let $p \in M$.

(1) We say that ϕ is smooth (or C^∞) at p if $\psi' \circ \phi \circ \psi^{-1}$ is smooth at $\psi(p)$ for some local coordinate functions $\psi : U \longrightarrow E \subseteq \mathbb{R}^n$, $\psi' : U' \to E' \subseteq \mathbb{R}^{n'}$ with $p \in U$, $\phi(p) \in U'$.
(2) We say that ϕ is a smooth map (or C^∞-map) if ϕ is smooth at all points of M.

Definition 2.7. [2] If $\phi : M \to M'$ satisfies the conditions (1) ϕ is bijective, (2) ϕ is smooth and (3) ϕ^{-1} is smooth, then we say that ϕ is a diffeomorphism and M, M' are diffeomorphic.

Definition 2.8. [14] Let V be a universal set. A neutrosophic set (NS) X in V is an object having the following form $X = \{(x, T_X(x), I_X(x), F_X(x)) \mid x \in V\}$, or $X : V \to [0,1] \times [0,1] \times [0,1]$ which is characterized by a truth-membership function T_X, an indeterminacy-membership function I_X and a falsity-membership function F_X. There is no restriction on the sum of $T_X(x)$, $I_X(x)$ and $F_X(x)$, therefore $0^- \leq \sup T_X(x) + \sup I_X(x) + \sup F_X(x) \leq 3^+$.

Definition 2.9. [13] Let X be a universal set. Then the $(\alpha,\ \beta,\ \gamma)$-cut neutrosophic set is denoted by $F^{(\alpha^+,\beta^+,\gamma^+)}$, where $\alpha, \beta, \gamma \in [0,1]$ and are fixed numbers such that $0 \leq \alpha + \beta + \gamma \leq 3$ is defined as $F^{(\alpha^+,\beta^+,\gamma^+)} = \{\{T_A(x),\ I_A(x),\ F_A(x)\} \mid x \in X, T_A(x) > \alpha, I_A(x) < \beta, F_A(x) < \gamma\}$.

Definition 2.10. (*George and Veeramani*) [2] A fuzzy metric space is an ordered triple $(X,\ M,\ *)$ such that X is a (nonempty) set, $*$is a continuous t-norm and M is a fuzzy set on $X \times X \times]\ \mathrm{O}, +\infty[$ satisfying the following conditions, for all $x, y, z \in X, s, t > 0$:
(GV1) $M(x,\ y,\ t) > 0$;
(GV2) $M(x,\ y,\ t) = 1$ if and only if $x = y$;
(GV3) $M(x,\ y,\ t) = M(y,\ x,\ t)$;
(GV4)$M(x,\ y,\ t) * M(y,\ z,\ s) \leq M(x,\ z,\ t+s)$; (GV5) $M(x,\ y,\ -) :]0, +\infty[\to]0, 1]$ is continuous.

Theorem 2.11. *[2] Let $f : X \to \mathbb{R}^+$ be a one-to-one function and let $g : \mathbb{R}^+ \to [0, +\infty[$ be an increasing continuous function. Fixed $\alpha, \beta > 0$, define M by*

$$M(x,\ y,\ t) = (\frac{(\min\{f(x), f(y)\})^\alpha + g(t)}{(\max\{f(x), f(y)\})^\alpha + g(t)})^\beta$$

Then, $(M,\ \cdot)$ is a fuzzy metric on X.

3. KM-single Valued Neutrosophic Topological Space

In this section, we introduce some definitions on KM-single valued neutrosophic topology.

Definition 3.1. Let M be an arbitrary nonempty set and $SVN(M) = \{(\mu, \nu, \varrho) : M^3 \to [0,1]^3\}$. A family $\mathcal{SVN}_\tau$ of subset of $SVN(M)$ is called a KM-single valued neutrosophic topology if it is satisfied in the following conditions:

(i) $(\mu,\nu,\varrho)_0 \in \mathcal{SVN}_\tau$ and $(\mu,\nu,\varrho)_1 \in \mathcal{SVN}_\tau$, where $(\mu,\nu,\varrho)_0(x) = (0,1,1)$, $(\mu,\nu,\varrho)_1(x) = (1,0,0)$, $0(x) = 0$ and $1(x) = 1$;

(ii) for every $i \in I$, if $(\mu_i,\nu_i,\varrho_i) \in \mathcal{SVN}_\tau$, then $(\bigcup_{i\in I}(\mu_i,\nu_i,\varrho_i)) \in \mathcal{SVN}_\tau$;

(iii) for every $1 \leq i \leq n$, if $(\mu_i,\nu_i,\varrho_i) \in \mathcal{SVN}_\tau$, then $(\bigcap_{i=1}^{n}(\mu_i,\nu_i,\varrho_i)) \in \mathcal{SVN}_\tau$.

We say that, $(M, \mathcal{SVN}_\tau)$ is a KM-single valued neutrosophic topological space and any member of $\mathcal{SVN}_\tau$ is an open KM-single valued neutrosophic subsets.

Example 3.2. *Consider $M = \mathbb{R}$ and*

$$\mathcal{SVN}_\tau = \{(\mu,\nu,\varrho)_1, (\mu_i,\nu_i,\varrho_i) \mid \textit{where}$$

$$(\mu_i,\nu_i,\varrho_i) = (\frac{i}{i+x^2}, 1-\frac{i^3}{i^3+\sqrt{x}}, 1-\frac{i^2}{i^2+e^x}) \textit{ and } i \in \mathbb{N}^*\}.$$

It is easy to see that $(M, \mathcal{SVN}_\tau)$ is a KM-single valued neutrosophic topological space.

Theorem 3.3. *Let $(M, \mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic topological space and $\alpha \in [0,1]$. Then $(M, \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)})$ is a topological space.*

Proof. Since $(\mu,\nu,\varrho)_0 \in \mathcal{SVN}_\tau$,we get that $(\mu,\nu,\varrho)_0^{(\alpha^+,\beta^+,\gamma^+)} = \{x \mid (\mu(x) > \alpha, \nu(x) < \beta, \varrho(x) < \gamma\} = \emptyset$ and so
$\emptyset \in \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)} = \{(\mu^{\alpha^+}, \nu^{\beta^+}, \varrho^{\gamma^+}) \mid (\mu,\nu,\varrho) \in \mathcal{SVN}_\tau\}$. In additon, $(\mu,\nu,\varrho)_1 \in \mathcal{SVN}_\tau$ implies that

$$(\mu,\nu,\varrho)_1^{(\alpha^+,\beta^+,\gamma^+)} = \{x \in X \mid \mu(x) > \alpha, \nu(x) < \beta, \varrho(x) < \gamma\} = X,$$

then $X \in \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)}$. Let $\{(\mu_i,\nu_i,\varrho_i)^{(\alpha^+,\beta^+,\gamma^+)}\}_{i\in I} \in \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)}$. Since

$$\bigvee_{i\in I}(\mu_i,\nu_i,\varrho_i)(x) = (\bigcup_{i\in I}(\mu_i,\nu_i,\varrho_i))(x) = (\bigcup_{i\in I}\mu_i(x), \bigcup_{i\in I}\nu_i(x), \bigcup_{i\in I}\varrho_i(x))$$

and for all $i \in I$, $\mu_i(x) > \alpha, \nu_i(x) < \beta, \varrho_i(x) < \gamma$, we have $\bigvee_{i\in I}\mu_i(x) > \alpha$, $\bigvee_{i\in I}\nu_i(x) < \beta$, $\bigvee_{i\in I}\varrho_i(x) < \gamma$ and so $\bigcup_{i\in I}(\mu_i,\nu_i,\varrho_i)^{(\alpha^+,\beta^+,\gamma^+)} \in \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)}$.
If $\{(\mu_i,\nu_i,\varrho_i)^{(\alpha^+,\beta^+,\gamma^+)}\}_{i=1}^{n} \in \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)}$, then

$$(\bigcap_{i=1}^{n}(\mu_i,\nu_i,\varrho_i))(x) = (\bigcap_{i=1}^{n}\mu_i(x), \bigcap_{i=1}^{n}\nu_i(x), \bigcap_{i=1}^{n}\varrho_i(x))$$

and for all $1 \leq i \leq n$, $\mu_i(x) > \alpha, \nu_i(x) < \beta, \varrho_i(x) < \gamma$, we have $\bigwedge_{i=1}^{n} \mu_i(x) > \alpha, \bigwedge_{i=1}^{n} \nu_i(x) < \beta, \bigwedge_{i=1}^{n} \varrho_i(x) < \gamma$ and so $\bigwedge_{i=1}^{n} (\mu_i, \nu_i, \varrho_i)^{(\alpha^+,\beta^+,\gamma^+)} \in \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)}$. It follows that $(X, \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)})$ is a topological space. □

Example 3.4. *Consider the* KM*-single valued neutrosophic topological space* $(M, \mathcal{SVN}_\tau)$ *in Example 3.2 and* $\alpha = \frac{1}{8}, \beta = \frac{1}{2}, \gamma = \frac{1}{2}$. *Then* $(\mu, \nu, \varrho)_0^{(\alpha^+,\beta^+,\gamma^+)} = \emptyset$, $(\mu, \nu, \varrho)_1^{(\alpha^+,\beta^+,\gamma^+)} = \mathbb{R}$, *and for* $i \geq 2$,

$$\begin{aligned}(\mu, \nu, \varrho)_i^{(\alpha^+,\beta^+,\gamma^+)} &= \{x \in M \mid \frac{i}{i + x^2} > \frac{1}{8}, 1 - \frac{i^3}{i^3 + \sqrt{x}} < \frac{1}{2}, 1 - \frac{i^2}{i^2 + e^x} < \frac{1}{2}\} \\ &= \{x \in M \mid -\sqrt{7i} < x < 2Lni\}.\end{aligned}$$

So $(\mathbb{R}, \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)})$ *is a topological space, where*

$$\mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)} = \{\emptyset, \mathbb{R}, (-\sqrt{7i}, Lni) \mid i \geq 2\}.$$

Theorem 3.5. *Let* M' *be a set where* $|M| = |M'|$ *and* $(M, \mathcal{SVN}_\tau)$ *be a* KM*-single valued neutrosophic topological space. Then there exists a* KM*-single valued neutrosophic topology* $\mathcal{SVN}'_\tau$ *on* M' *in such a way that* $(M', \mathcal{SVN}'_\tau)$ *is a* KM*-single valued neutrosophic topological space.*

Proof. Since $|M| = |M'|$, there is a bijection $\phi : M' \to M$. Consider $\mathcal{SVN}'_\tau = \{(\mu \circ \phi, \nu \circ \phi, \varrho \circ \phi)_i \mid (\mu, \nu, \varrho)_i \in \mathcal{SVN}_\tau\}$, clearly $(M', \mathcal{SVN}'_\tau)$ is a KM-single valued neutrosophic topological space. □

Corollary 3.6. *Let* M *be a set where* $|M| = |\mathbb{R}|$. *Then there exists a* KM*-single valued neutrosophic topology* $\mathcal{SVN}_\tau$ *on* M *in such a way that* $(M, \mathcal{SVN}_\tau)$ *is a* KM*-single valued neutrosophic topological space.*

Definition 3.7. Let $(M, \mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic topological space. A subfamily $\mathcal{SVNB}_\tau$ of $\mathcal{SVN}_\tau$ is called a base if (i), for all $x \in M$, we have $(\bigvee_{(\mu,\nu,\varrho) \in \mathcal{SVNB}_\tau} (\mu, \nu, \varrho))(x) = (1, 0, 0)$ and (ii), $(\mu_1, \nu_1, \varrho_1), (\mu_2, \nu_2, \varrho_2) \in \mathcal{SVNB}_\tau$ implies that $(\mu_1, \nu_1, \varrho_1) \cap (\mu_2, \nu_2, \varrho_2) \in \mathcal{SVNB}_\tau$.

Example 3.8. *Consider the* KM*-single valued neutrosophic topological space* $(M, \mathcal{SVN}_\tau)$ *in Example 3.2. Then*

$$\mathcal{SVNB}_\tau = \{(\mu, \nu, \varrho)_1, (\mu_i, \nu_i, \varrho_i) \mid \textit{where}$$

$$(\mu_i, \nu_i, \varrho_i) = (\frac{i^{0.5}}{i^{0.5} + x^2}, 1 - \frac{i^2}{i^2 + \sqrt{x}}, 1 - \frac{i}{i + e^x}) \textit{ and } i \in \mathbb{N}^*\}$$

is a base of $\mathcal{SVN}_\tau$.

Theorem 3.9. *Let $(M, \mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic topological space and $\mathcal{SVNB}_\tau$ be a base for $\mathcal{SVN}_\tau$. Then every element of $\mathcal{SVN}_\tau$ is inclused in union of elements of $\mathcal{SVNB}_\tau$.*

Proof. Let $(\mu, \nu, \varrho) \in \mathcal{SVN}_\tau$. Then for all $x \in M$, we have $(\mu, \nu, \varrho)(x) =$

$$= (\mu, \nu, \varrho)(x) \bigvee (1,0,0) = (\mu, \nu, \varrho)(x) \vee \Big(\bigvee_{(\mu_i, \nu_i, \varrho_i) \in \mathcal{SVNB}_\tau} (\mu_i, \nu_i, \varrho_i)(x) \Big)$$

$$\leq \bigvee_{(\mu_i, \nu_i, \varrho_i) \in \mathcal{SVNB}_\tau} (\mu_i, \nu_i, \varrho_i)(x).$$

It follows that $(\mu, \nu, \varrho) \subseteq \bigcup_{(\mu_i, \nu_i, \varrho_i) \in \mathcal{SVNB}_\tau} (\mu_i, \nu_i, \varrho_i)$. □

Definition 3.10. Let $(M, \mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic topological space and $\mathcal{SVNB}_\tau$ be a base for $\mathcal{SVN}_\tau$. Define $\langle \mathcal{SVNB}_\tau \rangle = \{(\mu', \nu', \varrho') \in \mathcal{SVN}_\tau |\ \exists\, (\mu, \nu, \varrho) \in \mathcal{SVN}_\tau,\ \mu \subseteq \mu', \nu \supseteq \nu', \varrho \supseteq \varrho'\}$ and it is called by generated KM-single valued neutrosophic topology by $\mathcal{SVNB}_\tau$.

Example 3.11. *Consider the KM-single valued neutrosophic topological space $(M, \mathcal{SVN}_\tau)$ in Example 3.2 and $\mathcal{SVNB}_\tau$ be a base for $\mathcal{SVN}_\tau$ in Example 3.8. Then $\langle \mathcal{SVNB}_\tau \rangle = \{(\mu', \nu', \varrho') \in \mathcal{SVN}_\tau |\ \exists\, (\mu, \nu, \varrho) \in \mathcal{SVN}_\tau,\ \mu \subseteq \mu', \nu \supseteq \nu', \varrho \supseteq \varrho'\}$.*

Theorem 3.12. *Let $(M, \mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic topological space. Then $\langle \mathcal{SVNB}_\tau \rangle$ is a KM-single valued neutrosophic topology on M.*

Proof. Since for all $(\mu, \nu, \varrho) \in \mathcal{SVN}_\tau, (\mu, \nu, \varrho) \subseteq (\mu, \nu, \varrho)_1$, we get that $(\mu, \nu, \varrho)_1 \in \langle \mathcal{SVNB}_\tau \rangle$. If $(\mu, \nu, \varrho)_0 \in \langle \mathcal{SVNB}_\tau \rangle$, then $(\mu, \nu, \varrho) \subseteq (\mu, \nu, \varrho)_0$ implies that $(\mu, \nu, \varrho) = (\mu, \nu, \varrho)_0$. Let $\{(\mu_i, \nu_i, \varrho_i)\}_{i \in I}$ be a family of elements $\langle \mathcal{SVNB}_\tau \rangle$ and $\bigcup_{i \in I} (\mu_i, \nu_i, \varrho_i) = (\mu, \nu, \varrho)$. Then there exist $(\mu'_i, \nu'_i, \varrho'_i) \in \mathcal{SVN}_\tau$ in such a way that $\mu_i \subseteq \mu'_i, \nu_i \supseteq \nu'_i, \varrho_i \supseteq \varrho'_i$. So $\mu \subseteq \bigcup_{i \in I} \mu', \nu \supseteq \bigcup_{i \in I} \nu', \varrho \supseteq \bigcup_{i \in I} \varrho'$. Because$(\mu, \nu, \varrho) \in \mathcal{SVN}_\tau$ and $\mu \subseteq \mu', \nu \supseteq \nu', \varrho \supseteq \varrho'$, we get that $(\mu, \nu, \varrho) \in \langle \mathcal{SVNB}_\tau \rangle$. Now, if for $n \in \mathbb{N}$, $\{(\mu_i, \nu_i, \varrho_i)\}_{i=1}^n$ is a family of elements of $\langle \mathcal{SVNB}_\tau \rangle$ in a similar way we have $\bigcap_{i=1}^{n} (\mu_i, \nu_i, \varrho_i) \in \langle \mathcal{SVNB}_\tau \rangle$. □

Proposition 3.13. *Let $(M, \mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic topological space and $\mathcal{SVNB}_\tau$ be a base for $(M, \mathcal{SVN}_\tau)$. Then $\mathcal{SVNB}_\tau^{(\alpha^+, \beta^+, \gamma^+)}$ is a base for topological space $(M, \mathcal{SVN}_\tau^{(\alpha^+, \beta^+, \gamma^+)})$.*

Proof. Let $\mathcal{B} = \{(\mu, \nu, \varrho)^{(\alpha^+, \beta^+, \gamma^+)} |\ (\mu, \nu, \varrho) \in \mathcal{SVNB}_\tau, \alpha, \beta, \gamma \in [0,1]\},\ x \in M$ and $(\mu, \nu, \varrho)(x) = (\alpha, \beta, \gamma)$. Then $x \in (\mu, \nu, \varrho)^{(\alpha^+, \beta^+, \gamma^+)} \subseteq \bigcup_{\alpha, \beta, \gamma} (\mu, \nu, \varrho)^{(\alpha^+, \beta^+, \gamma^+)}$

and so $\bigcup_{\alpha,\beta,\gamma}(\mu,\nu,\varrho)^{(\alpha^+,\beta^+,\gamma^+)} = M$. Suppose $x \in (\mu_1,\nu_1,\varrho_1)^{(\alpha^+,\beta^+,\gamma^+)} \cap (\mu_2,\nu_2,\varrho_2)^{(\alpha^+,\beta^+,\gamma^+)}$. Since $\mathcal{SVNB}_\tau$ is a base for KM-single valued neutrosophic topological space $(M,\mathcal{SVN}_\tau)$, we get $(\mu_1,\nu_1,\varrho_1)\cap(\mu_2,\nu_2,\varrho_2) \in \mathcal{SVNB}_\tau$. Now, $x \in ((\mu_1,\nu_1,\varrho_1)\cap(\mu_2,\nu_2,\varrho_2))^{(\alpha^+,\beta^+,\gamma^+)} \subseteq$
$(\mu_1,\nu_1,\varrho_1)^{(\alpha^+,\beta^+,\gamma^+)} \cap (\mu_2,\nu_2,\varrho_2)^{(\alpha^+,\beta^+,\gamma^+)}$ and so $\mathcal{B}$ is a base for $\mathcal{SVNB}_\tau$. □

In the following, we will construct KM-single valued neutrosophic topological space via topological spaces.

Definition 3.14. Let $(M^3,(\tau_M,\tau'_M,\tau''_M))$ be a topological space. For all $M_i \in \tau_M$, define $(\tau,\tau',\tau'')_\emptyset = (\mu,\nu,\varrho)_0, (\tau_M,\tau'_M,\tau''_M) = (\mu,\nu,\varrho)_1$ and for any $i \in I, (\tau_{M_i},\tau'_{M_i},\tau''_{M_i}) : M^3 \to [0,1]$ by $(\tau_{M_i},\tau'_{M_i},\tau''_{M_i})(x) = (\mu_i,\nu_i,\varrho_i)(x) = (\alpha_i,\beta_i,\gamma_i) \in [0,1]$.

Example 3.15. *Let* $M = \{a,b,c\}$, $\tau_M = \{\emptyset,\{a\},\{b,c\},M\}$, $\tau'_M = \{\emptyset,\{b\},\{a,c\},M\}$, $\tau_M = \{\emptyset,\{a\},\{b\},\{c\},M\}$. *Define* $(\tau,\tau',\tau'')_\emptyset = (\mu,\nu,\varrho)_0, (\tau_M,\tau'_M,\tau''_M) = (\mu,\nu,\varrho)_1$, $(\tau(a) = 0.1, \tau'(a) = 0.2, \tau''(a) = 0.3, \tau(b) = 0.15, \tau'(b) = 0.18, \tau''(b) = 0.31, \tau(c) = 0.55, \tau'(c) = 0.16, \tau''(c) = 0.9$ *and for* $S \subseteq \tau_M, S' \subseteq \tau'_M, S'' \subseteq \tau''_M$, $\tau(S) = \bigwedge_{x\in S}\tau(x), \tau'(S') = \bigvee_{x\in S'}\tau'(x)$ *and* $\tau''(S'') = \bigwedge_{x\in S}\tau''(x)$.

Theorem 3.16. *Let* $(M^3,(\tau_M,\tau'_M,\tau''_M))$ *be a topological space. Then* $(M^3,\mathcal{SVN}_\tau = \{(\tau_{M_i},\tau'_{M_i},\tau''_{M_i})\}_{i\in I})$ *is a KM-single valued neutrosophic topological space.*

Proof. Since $(M^3,(\tau_M,\tau'_M,\tau''_M))$ is a topological space, we have $\emptyset, M \in \tau_M$ so by definition

$$(\mu,\nu,\varrho)_0 = (\tau,\tau',\tau'')_\emptyset, (\mu,\nu,\varrho)_0 = (\tau_M,\tau'_M,\tau''_M) \in \mathcal{SVN}_\tau.$$

Let for any $i \in I$, we have $(\mu_i,\nu_i,\varrho_i) \in \mathcal{SVN}_\tau$. Since for all $x \in M$,

$$\begin{aligned}(\bigcup_{i\in I}(\mu_i,\nu_i,\varrho_i))(x) &= \bigvee_{i\in I}(\mu_i,\nu_i,\varrho_i)(x) = \bigvee_{M_i\in\tau_M}(\tau_{M_i},\tau'_{M_i},\tau''_{M_i})(x) \\ &= (\bigcup_{M_i\in\tau_M}(\tau_{M_i},\tau'_{M_i},\tau''_{M_i}))(x)\end{aligned}$$

and $(M^3,(\tau_M,\tau'_M,\tau''_M))$ is a topological space, we get $\bigcup_{M_i\in\tau_M}(\tau_{M_i},\tau'_{M_i},\tau''_{M_i}) \in \tau_M$ and so $\bigcup_{i\in I}(\mu_i,\nu_i,\varrho_i) \in \mathcal{SVN}_\tau$. Let $n \in \mathbb{N}$ and $\{(\mu_i,\nu_i,\varrho_i)\}_{i=1}^n$ be a set of elements $\mathcal{SVN}_\tau$. In a similar way, it is shown that $\bigcap_{i=1}^n(\mu_i,\nu_i,\varrho_i) \in \mathcal{SVN}_\tau$.

Therefore, $(M^3, \mathcal{SVN}_\tau)$ is a KM-single valued neutrosophic topological space. □

Corollary 3.17. *Let M be a non-empty set. Then there exists a KM-single valued neutrosophic topology $\mathcal{SVN}_\tau$ on M such that $(M^3, \mathcal{SVN}_\tau)$ is a KM-single valued neutrosophic topological space.*

Definition 3.18. Let $(M^3, \mathcal{SVN}_\tau)$ and $(M'^3, \mathcal{SVN}_{\tau'})$ be KM-single valued neutrosophic topological spaces and $f : (M^3, \mathcal{SVN}_\tau) \to (M'^3, \mathcal{SVN}_{\tau'})$ be a homeomorphism, define

$$f^{(\alpha^+,\beta^+,\gamma^+)} : (M^3, \mathcal{SVN}_\tau^{(\alpha^+,\beta^+,\gamma^+)}) \to (M'^3, (\mathcal{SVN}_{\tau'})^{(\alpha^+,\beta^+,\gamma^+)})$$

by $f^{(\alpha^+,\beta^+,\gamma^+)}(x) = f(x)$, where $x \in M$.

Example 3.19. *Consider $M = \mathbb{R}$ and*

$$\mathcal{SVN}_\tau = \{(\mu, \nu, \varrho)_1, (\mu_i, \nu_i, \varrho_i) \mid \textit{where}$$

$$(\mu_i, \nu_i, \varrho_i) = (\frac{i}{i + x^2}, 1 - \frac{i^3}{i^3 + \sqrt{x}}, 1 - \frac{i^2}{i^2 + e^x}) \textit{ and } i \in \mathbb{N}^*\}$$

and

$$\mathcal{SVN}'_\tau = \{(\mu', \nu', \varrho')_1, (\mu'_i, \nu'_i, \varrho'_i) \mid \textit{where}$$

$$(\mu'_i, \nu'_i, \varrho'_i) = (\frac{i^2}{i^2 + x^2}, 1 - \frac{i^6}{i^6 + \sqrt{x}}, 1 - \frac{i^4}{i^4 + e^x}) \textit{ and } i \in \mathbb{N}^*\}.$$

It is easy to see that $(M, \mathcal{SVN}_\tau)$ and $(M, \mathcal{SVN}'_\tau)$ are KM-single valued neutrosophic topological spaces. Define

$$f^{(\alpha^+,\beta^+,\gamma^+)} : (M^3, \mathcal{SVN}_\tau^{(0.5^+,0.6^+,0.7^+)}) \to (M'^3, (\mathcal{SVN}_{\tau'})^{(0.5^+,.6^+,0.7^+)})$$

by $f^{(0.5^+,0.6^+,0.7^+)}(x) = f(x)$, where $x \in M$.

From on now, we introduce a concept of KM-single valued neutrosophic Hausdorff space based on valued-cuts and in this regards, the concept of KM-single valued neutrosophic manifold space is presented.

Definition 3.20. Let $(M, \mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic topological space. Then $(M, \mathcal{SVN}_\tau)$ is called a KM-single valued neutrosophic Hausdorff space if, for all $x, y \in M$ there exist $(\mu_1, \nu_1, \varrho_1), (\mu_2, \nu_2, \varrho_2) \in \mathcal{SVN}_\tau$ and $0 < \alpha < \beta < 1, 0 < \alpha' < \beta' < 1, 0 < \alpha'' < \beta'' < 1$ in such a way that

$$x \in (\mu_1, \nu_1, \varrho_1)^{((\alpha,\beta)^+,(\alpha',\beta')^+,(\alpha'',\beta'')^+)}, y \in (\mu_2, \nu_2, \varrho_2)^{((\alpha,\beta)^+,(\alpha',\beta')^+,(\alpha'',\beta'')^+)}$$

and

$$(\mu_1, \nu_1, \varrho_1)^{((\alpha,\beta)^+,(\alpha',\beta')^+,(\alpha'',\beta'')^+)} \cap (\mu_2, \nu_2, \varrho_2)^{((\alpha,\beta)^+,(\alpha',\beta')^+,(\alpha'',\beta'')^+)} = \emptyset,$$

where for all $(\mu, \nu, \varrho) \in \mathcal{SVN}_\tau$, we have

$$(\mu, \nu, \varrho)^{((\alpha,\beta)^+,(\alpha',\beta')^+,(\alpha'',\beta'')^+)} = \{x \in M \mid (\alpha, \alpha', \alpha'') < (\mu, \nu, \varrho)(x) < (\beta, \beta', \beta'')\}.$$

Example 3.21. *Consider the KM-single valued neutrosophic topological space, which is defined in Example 3.2. A simple computations show that for $i \neq i', j \neq j', m \neq m' \in \mathbb{N}^*$*

$$J_i = (-\sqrt{i(\frac{1}{\alpha}-1)}, -\sqrt{i(\frac{1}{\beta}-1)}) \cup (\sqrt{i(\frac{1}{\beta}-1)}, \sqrt{i(\frac{1}{\alpha}-1)})$$
$$J_{i'} = (-\sqrt{i'(\frac{1}{\alpha}-1)}, -\sqrt{i'(\frac{1}{\beta}-1)}) \cup (\sqrt{i'(\frac{1}{\beta}-1)}, \sqrt{i'(\frac{1}{\alpha}-1)}),$$
$$K_i = ((\frac{\alpha' i^3}{1-\alpha'})^2, (\frac{\beta' i^3}{1-\beta'})^2), K_{i'} = ((\frac{\alpha' i'^3}{1-\alpha'})^2, (\frac{\beta' i'^3}{1-\beta'})^2)$$
$$L_i = (Ln(\frac{i^2\alpha''}{1-\alpha''}), Ln(\frac{i^2\beta''}{1-\beta''})), L_{i'} = (Ln(\frac{i'^2\alpha''}{1-\alpha''}), Ln(\frac{i'^2\beta''}{1-\beta''}))$$

where $J_i = \mu_i^{(\alpha,\beta)^+}$, $J_{i'} = \mu_{i'}^{(\alpha,\beta)^+}$, $K_i = \nu_i^{(\alpha',\beta')^+}$, $K_{i'} = \nu_{i'}^{(\alpha',\beta')^+}$, $L_i = \varrho_i^{(\alpha'',\beta'')^+}$ and $L_{i'} = \varrho_{i'}^{(\alpha'',\beta'')^+}$. If $x, y \in \mathbb{R}$, since $\mathbb{R}$ is a Hausdorff space, there exists $i, i', j, j', m, m' \in \mathbb{N}^*$ such that $x \in J_i$, $y \in J_{i'}$,$x \in K_j$, $y \in K_{j'}$,$x \in L_m$, $y \in L_{m'}$, and $J_i \cap J_{i'} = \emptyset$,$K_j \cap K_{j'} = \emptyset$,$L_m \cap L_{m'} = \emptyset$. Since for all $x \in \mathbb{R}$, if consider $0 \leq \alpha < \beta \leq 1$, then

$$\frac{\alpha x^2}{1-\alpha} < i < \frac{\beta x^2}{1-\beta},$$
$$(\frac{\sqrt{x}(1-\beta'}{\beta'})^{\frac{1}{3}} < j < \frac{\sqrt{x}(1-\alpha'}{\alpha'})^{\frac{1}{3}},$$
$$\{-\sqrt{\frac{(1-\alpha'')e^x}{\alpha'}} < m < -\sqrt{\frac{(1-\beta'')e^x}{\beta''}}\} \cup$$
$$\{\sqrt{\frac{(1-\beta'')e^x}{\beta''}} < m < \sqrt{\frac{(1-\alpha'')e^x}{\alpha'}}.$$

Theorem 3.22. *Let $(M, \mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic Hausdorff space and $\alpha, \beta, \gamma \in [0,1]$. Then $(M, \mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)})$ is a Hausdorff space and conversely.*

Proof. Since $(M, \mathcal{SVN}_\tau)$ is a KM-single valued neutrosophic topological space, by Theorem 3.3, $(M, \mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)})$ is a topological space. Let $x, y \in M$ and $\gamma, \delta, \gamma', \delta', \gamma'', \delta'' \in [0,1]$. Since $(M, \mathcal{SVN}_\tau)$ is a KM-single valued neutrosophic Hausdorff space, there exist, $(\mu_1, \nu_1, \varrho_1), (\mu_2, \nu_2, \varrho_2) \in \mathcal{SVN}_\tau$ such that $x \in (\mu_1, \nu_1, \varrho_1)^{((\gamma,\delta)^+,(\gamma',\delta')^+,(\gamma'',\delta'')^+)}$ and $y \in (\mu_2, \nu_2, \varrho_2)^{((\zeta,\eta)^+,(\zeta',\eta')^+,(\zeta'',\eta'')^+)}$. Now consider $\alpha \leq T_{min}\{\gamma, \delta\}, \beta \leq T_{min}\{\gamma', \delta'\}, \gamma \leq T_{min}\{\gamma'', \delta''\}$. It follows that $x \in (\mu_1, \nu_1, \varrho_1)^{(\alpha,\beta,\gamma)}$, $y \in (\mu_2, \nu_2, \varrho_2)^{(\alpha,\beta,\gamma)}$ and $(\mu_1, \nu_1, \varrho_1)^{(\alpha,\beta,\gamma)} \cap (\mu_2, \nu_2, \varrho_2)^{(\alpha,\beta,\gamma)} = \emptyset$. Therefore $(M, \mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)})$ is a Haussdorff space.

Let $\alpha, \beta, \gamma \in [0,1]$. Since $(M, \mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)})$ is a Hausdorff space, we get that $\emptyset \in \mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)}$. It follows that there exists $(\mu, \nu, \varrho) \in \mathcal{SVN}_\tau$ in such a way that

$(\mu,\nu,\varrho)^{(\alpha^+,\beta^+,\gamma^+)}=\emptyset$. Thus for any $x\in M, \mu(x)\leq\alpha, \nu(x)\geq\beta$ and $\varrho(x)\geq\gamma$. It follows that $(\mu,\nu,\varrho)_0\in\mathcal{SVN}_\tau$. In addition, $X\in\mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)}$ follows that there is $(\mu,\nu,\varrho)\in\mathcal{SVN}_\tau$, such that $(\mu,\nu,\varrho)^{(\alpha^+,\beta^+,\gamma^+)}=X$. Thus for any $x\in M, \mu(x)\geq\alpha, \nu(x)\leq\beta$ and $\varrho(x)\leq\gamma$. It follows that $(\mu,\nu,\varrho)_1\in\mathcal{SVN}_\tau$. Let for every $i\in I$, if $(\mu_i,\nu_i,\varrho_i)\in\mathcal{SVN}_\tau$. Since $(M,\mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)})$ is a Hausdorff space and $(\mu_i,\nu_i,\varrho_i)^{(\alpha,\beta,\gamma)}\in\mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)}$, we get that $\bigcup_{i\in I}(\mu_i,\nu_i,\varrho_i)^{(\alpha,\beta,\gamma)}\in\mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)}$. It follows that for $x\in M, \mu_i(x)\geq\alpha, \nu_i(x)\geq\beta$ and $\varrho_i(x)\geq\gamma$. Thus $\bigvee_{i\in I}\mu_i(x)\geq\alpha, \bigvee_{i\in I}\nu_i(x)\leq\beta$ and $\bigvee_{i\in I}\varrho_i(x)\leq\gamma$. So $(\bigcup_{i\in I}(\mu_i,\nu_i,\varrho_i))\in\mathcal{SVN}_\tau$. In similar to, other axioms are valied, then $(M,\mathcal{SVN}_\tau)$ is a KM-single valued neutrosophic Hausdorff space. □

Definition 3.23. Let $(M,\mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic Hausdorff space. So $(M,\mathcal{SVN}_\tau)$ is called a KM-single valued neutrosophic manifold if, for all $x\in M$, there exists $(\mu,\nu,\varrho)\in\mathcal{SVN}_\tau$ and a homeomorphism $\phi: supp((\mu,\nu,\varrho))\to\mathbb{R}^n$ such that $x\in Supp((\mu,\nu,\varrho))$. Each $((\mu,\nu,\varrho),\phi)$ is called a KM-single valued neutrosophic chart and $\mathcal{A}=\{((\mu,\nu,\varrho),\phi)\mid(\mu,\nu,\varrho)\in\mathcal{SVN}_\tau,\ \phi: supp((\mu,\nu,\varrho))\to\mathbb{R}^n\}$ is called a KM-single valued neutrosophic atlas. Let $((\mu,\nu,\varrho),\phi),((\mu',\nu',\varrho'),\psi)$ be two KM-single valued neutrosophic charts of KM-single valued neutrosophic atlas $\mathcal{A}$. Then
$((\mu,\nu,\varrho),\phi),((\mu',\nu',\varrho'),\psi)$ are called C^∞-compatible charts if

$$\phi: supp((\mu,\nu,\varrho))\to\mathbb{R}^n, \psi: supp(\mu',\nu',\varrho')\to\mathbb{R}^n$$

and

$$\phi\circ\psi^{-1}:\psi(Supp((\mu,\nu,\varrho))\cap Supp(\mu',\nu',\varrho'))\to\phi(Supp((\mu,\nu,\varrho))\cap Supp(\mu',\nu',\varrho'))$$

are a C^1-KM-single valued neutrosophic diffeomorphism.

Example 3.24. *Consider the KM-single valued neutrosophic Hausdorff space, which is defined in Example 3.21. It is clear that for all $x\in\mathbb{R}$, and for all $i\in\mathbb{N}$, we have $x\in Supp((\mu_i,\nu_i,\varrho_i))=\mathbb{R}$, we get that $x\in Supp((\mu_i,\nu_i,\varrho_i))$. Define $(Ln)_i: Supp((\mu_i,\nu_i,\varrho_i))\longrightarrow\mathbb{R}$, so $\mathcal{A}=\{((\mu_i,\nu_i,\varrho_i),(Ln)_i)\mid i\in\mathbb{N}\}$ is a KM-single valued neutrosophic atlas.*

Theorem 3.25. *Let $(M,\mathcal{SVN}_\tau)$ be a KM-single valued neutrosophic manifold and $\alpha,\beta,\gamma\in[0,1]$. Then $(M,\mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)})$ is a manifold and converesly.*

Proof. Since $(M,\mathcal{SVN}_\tau)$ is a KM-single valued neutrosophic manifold, Theorem 3.22, implies that $(M,\mathcal{SVN}_\tau^{(\alpha,\beta,\gamma)})$ is a Hausdorff topological space. In addition, for all $x\in M$, there exists $(\mu,\nu,\varrho)\in\mathcal{SVN}_\tau$ and homeomorphism $\phi: supp((\mu,\nu,\varrho))\to\mathbb{R}^n$ such that $x\in Supp((\mu,\nu,\varrho))$. Now consider $\mathcal{A}^\alpha=\{((\mu,\nu,\varrho)^\alpha,\phi)\mid(\mu,\nu,\varrho)\in\mathcal{SVN}_\tau,\ \phi: supp((\mu,\nu,\varrho))\to\mathbb{R}^n\}$. One can see that $\mathcal{A}^\alpha$ is an KM-single valued neutrosophic topological space atlas. Thus, $(M,\mathcal{SVN}_\tau^\alpha)$ is a manifold.

The converse is similar to the proof of Theorem 3.22. □

In the following, we want to extend two KM-single valued neutrosophic topological space to a larger class of KM-single valued neutrosophic topological spaces.

Definition 3.26. Let $\mathcal{SVN}_\tau$ and $\mathcal{SVN}_{\tau'}$ be KM-single valued neutrosophic topology on M and M', respectively. Define $\mathcal{SVN}_\tau \times \mathcal{SVN}_{\tau'} = \{(\mu,\nu,\varrho)\times(\mu',\nu',\varrho') \mid (\mu,\nu,\varrho)\in\mathcal{SVN}_\tau, (\mu',\nu',\varrho')\in\mathcal{SVN}_{\tau'}\}$, where for all $x,y\in M\times M'$ and $((\mu,\nu,\varrho)\times(\mu',\nu',\varrho'))(x,y) = T_{min}((\mu,\nu,\varrho)(x),(\mu',\nu',\varrho')(y))$.

Example 3.27. *Consider the KM-single valued neutrosophic topological space $(M,\mathcal{SVN}_\tau)$ in Example 3.19. Then*
$\mathcal{SVN}_\tau \times \mathcal{SVN}_{\tau'} = \{(\mu,\nu,\varrho)\times(\mu',\nu',\varrho') \mid (\mu,\nu,\varrho)\in\mathcal{SVN}_\tau, (\mu',\nu',\varrho')\in \mathcal{SVN}_{\tau'}\}$*, where for all $x,y\in M\times M'$ and*
$((\mu,\nu,\varrho)\times(\mu',\nu',\varrho'))(x,y) =$
$T_{min}((\frac{i}{i+x^2},1-\frac{i^3}{i^3+\sqrt{x}},1-\frac{i^2}{i^2+e^x}(x),(\frac{i^2}{i^2+x^2},1-\frac{i^6}{i^6+\sqrt{x}},1-\frac{i^4}{i^4+e^x})(y))$.

Theorem 3.28. *Let $(M,\mathcal{SVN}_\tau)$ and $(M',\mathcal{SVN}_{\tau'})$ be KM-single valued neutrosophic topological spaces. Then $(M\times M',\mathcal{SVN}_\tau\times\mathcal{SVN}_{\tau'})$ is a KM-single valued neutrosophic topological space and conversely.*

Proof. Since $(\mu,\nu,\varrho)_0\in\mathcal{SVN}_\tau\cap\mathcal{SVN}_{\tau'}$ for all $(x,y)\in M\times M'$ we have

$$((\mu,\nu,\varrho)_0\times(\mu,\nu,\varrho)_0)(x,y) = T_{min}\{(\mu,\nu,\varrho)_0(x),(\mu,\nu,\varrho)_0(y)\} = 0,$$

and so $((\mu,\nu,\varrho)_0\times(\mu,\nu,\varrho)_0)\in\mathcal{SVN}_\tau\times\mathcal{SVN}_{\tau'}$. In addition, $(\mu,\nu,\varrho)_1\in\mathcal{SVN}_\tau\cap\mathcal{SVN}_{\tau'}$, implies that

$$((\mu,\nu,\varrho)_1\times(\mu,\nu,\varrho)_1)(x,y) = T_{pr}\{(\mu,\nu,\varrho)_1(x),(\mu,\nu,\varrho)_1(y)\} = 1$$

and so $(\mu,\nu,\varrho)_1\times(\mu,\nu,\varrho)_1\in\mathcal{SVN}_\tau\times\mathcal{SVN}_{\tau'}$. Let
$\{(\mu_i,\nu_i,\varrho_i)\}_{i\in I}$ and$\{(\mu'_j,\nu'_j,\varrho'_j)\}_{j\in J}$ be two families of KM-single valued neutrosophictopologies on M and M', respectively. Then

$$\begin{aligned}
&(\bigcup_{i\in I,j\in J}((\mu_i,\nu_i,\varrho_i)\times(\mu'_j,\nu'_j,\varrho'_j)))(x,y)\\
&=(\bigvee_{i\in I,j\in J}((\mu_i,\nu_i,\varrho_i)\times(\mu'_j,\nu'_j,\varrho'_j)))(x,y)\\
&=\bigvee_{i\in I,j\in J}(T_{min}((\mu_i,\nu_i,\varrho_i)\times(\mu'_j,\nu'_j,\varrho'_j)))
\end{aligned}$$

implies that $\bigcup_{i\in I,j\in J}((\mu_i,\nu_i,\varrho_i)\times(\mu'_j,\nu'_j,\varrho'_j))\in\mathcal{SVN}_\tau\times\mathcal{SVN}_{\tau'}$.
Let $\{(\mu_i,\nu_i,\varrho_i)\}_{i=1}^n$ and $\{(\mu'_j,\nu'_j,\varrho'_j)\}_{j=1}^m$ be two families of KM-single valued

neutrosophictopologies on M and M',respectively. Then

$$\bigcap_{i=1,j=1}^{n,m}((\mu_i,\nu_i,\varrho_i)\times(\mu'_j,\nu'_j,\varrho'_j)))(x,y) = (\bigvee_{i=1,j=1}^{n,m}((\mu_i,\nu_i,\varrho_i)\times(\mu'_j,\nu'_j,\varrho'_j)))(x,y)$$
$$= \bigvee_{i=1,j=1}^{n,m}(T_{min}((\mu_i,\nu_i,\varrho_i),(\mu'_j,\nu'_j,\varrho'_j)))$$

implies that $\bigcap_{i=1,j=1}^{n,m}((\mu_i,\nu_i,\varrho_i)\times(\mu'_j,\nu'_j,\varrho'_j)) \in \mathcal{SVN}_\tau \times \mathcal{SVN}_{\tau'}$. Thus $(M \times M', \mathcal{SVN}_\tau \times \mathcal{SVN}_{\tau'})$ is a KM-single valued neutrosophic topological space.

The Converese of Theorem is similar to. □

4. Conclusion

The current paper has introduced a novel concept of KM-single valued neutrosophic topological space. Indeed it has presented a new generalization of fuzzy topological space. This work extended and obtained some properties by valued cuts based on the KM-single valued neutrosophic topological space. We hope that these results are helpful for further studies in geometry theory. In our future studies, we hope to obtain more results regarding to hypergeometry theory as a generalization of geometry theory based on fuzzy subset and its generalization and obtain some results in this regard to husdorf spaces and their applications.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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