Journal of Mahani Mathematical Research
Shahid Bahonar University of Kerman
Print ISSN: 2251-7952 Online ISSN: 2645-4505

THE COMPLETE CLASSIFICATION OF QUARTER-SYMMETRIC MAGNETIC CURVES IN S-MANIFOLDS

Ş. Güvenç

Article type: Research Article
(Received: 25 April 2022, Received in revised form: 17 June 2022)
(Accepted: 03 July 2022, Published Online: 03 July 2022)

Abstract. In this paper, we consider S-manifolds endowed with a quarter-symmetric metric connection. We obtain the condition for a curve to be magnetic with respect to this connection. We show that quarter-symmetric magnetic curves are $\theta_\alpha-$slant curves of osculating order $r \leq 3$ with constant quarter-symmetric curvature functions. Finally, we give the classification theorem.

Keywords: Magnetic curve, $\theta_\alpha-$slant curve, S-manifold, Quarter-symmetric metric connection.
2020 MSC: Primary 53C25, 53C40, 53A04.

1. Introduction

Let (M, g) be a $(2n+s)$-dimensional Riemann manifold. M is called *framed metric $\varphi-$manifold* **[21]** with a *framed metric structure* $(\varphi, \xi_\alpha, \eta^\alpha, g)$, $\alpha \in \{1, ..., s\}$, if this structure satisfies the following equations:

$$\varphi^2 = -I + \sum_{\alpha=1}^{s} \eta^\alpha \otimes \xi_\alpha, \quad \eta^\alpha(\xi_\beta) = \delta^\alpha_\beta,$$

$$\varphi(\xi_\alpha) = 0, \quad \eta^\alpha \circ \varphi = 0,$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \sum_{\alpha=1}^{s} \eta^\alpha(X)\eta^\alpha(Y),$$

$$d\eta^\alpha(X, Y) = g(X, \varphi Y) = -d\eta^\alpha(Y, X), \quad \eta^\alpha(X) = g(X, \xi_\alpha),$$

where, φ is a $(1,1)$ tensor field of rank $2n$; $\xi_1, ..., \xi_s$ are vector fields; $\eta^1, ..., \eta^s$ are 1-forms and g is a Riemannian metric on M; $X, Y \in \chi(M)$ and $\alpha, \beta \in \{1, ..., s\}$.

$(\varphi, \xi_\alpha, \eta^\alpha, g)$ is said to be an *S-structure*, if the Nijenhuis tensor of φ is equal to $-2d\eta^\alpha \otimes \xi_\alpha$, where $\alpha \in \{1, ..., s\}$ **[2]**. In this case, $(M, \varphi, \xi_\alpha, \eta^\alpha, g)$ is called an *S-manifold*. When $s = 1$, a framed metric structure turns into an almost

✉ sguvenc@balikesir.edu.tr, ORCID: 0000-0001-6254-4693
DOI: 10.22103/jmmr.2022.19400.1244

Publisher: Shahid Bahonar University of Kerman
How to cite: Ş. Güvenç, *The Complete Classification of Quarter-Symmetric Magnetic Curves in S-Manifolds*, J. Mahani Math. Res. 2023; 12(1): 151-160.

contact metric structure and an S-structure turns into a Sasakian structure. For an S-structure, the following equations are satisfied **[2], [5]**:

$$(\nabla_X \varphi)(Y) = \sum_{\alpha=1}^{s} \left\{ g(\varphi X, \varphi Y)\xi_\alpha + \eta^\alpha(Y)\varphi^2 X \right\}, \tag{1}$$

$$\nabla_X \xi_\alpha = -\varphi X, \;\; \alpha \in \{1, ..., s\}. \tag{2}$$

If M is Sasakian ($s = 1$), (2) can be directly calculated from (1). Note that the term "S-manifold" has been used by A. J. Ledger and others as a generalisation of E. Cartan's symmetric spaces, e.g. [15], [16] and [19].

Now, let us consider an S-manifold endowed with a quarter-symmetric metric connection and obtain a condition (conditions) under which a curve is magnetic with respect to this connection in the sense of Ozgur and Guvenc (the present author).

Definition 1.1. A pseudo-Hermitian magnetic curve in a Sasakian manifold is a curve satisfying one of the following equations:

- Ozgur and Guvenc's sense: **[12]**

$$\widehat{\nabla}_T T = (-q + 2\cos\theta)\,\varphi T,$$

- Lee's sense: **[17]**

$$\widehat{\nabla}_T T = -q\varphi T,$$

where $\widehat{\nabla}$ is the Tanaka-Webster connection, q is non-zero constant, θ is the contact angle of the curve and T is the unit tangent vector field along the curve. Here, the minus sign $(-)$ is for orientation purposes.

It is a straightforward motivation to replace another metric connection and study in the same direction of the above papers. For this achievement, we will choose the following quarter-symmetric metric connection in an S-manifold: **[9]**

$$\overline{\nabla}_X Y = \nabla_X Y + \sum_{\alpha=1}^{s} \eta_\alpha (X)\,\varphi Y, \tag{3}$$

where ∇ is the Levi-Civita connection and $X, Y \in \chi(M)$.

1.1. **Magnetic curves.**

Definition 1.2. Let (M, g) be a Riemannian manifold, F a closed 2-form and let us denote the Lorentz force on M by Φ, which is a $(1,1)$-type tensor field. If F is associated by the relation

$$g(\Phi X, Y) = F(X, Y), \;\; \forall X, Y \in \chi(M),$$

then it is called a *magnetic field* (**[1]**, **[4]** and **[6]**).

Let ∇ be the Riemannian connection associated to the Riemannian metric g and $\gamma : I \to M$ a smooth curve. If γ satisfies the Lorentz equation

$$\nabla_{\gamma'(t)}\gamma'(t) = \Phi(\gamma'(t)), \tag{4}$$

then it is called a *magnetic curve* or a *trajectory* for the magnetic field F.

The Lorentz equation is a generalization of the equation for geodesics. A curve which satisfies the Lorentz equation is called *magnetic trajectory.* Magnetic trajectories have constant speed. If the speed of the magnetic curve γ is equal to 1, then it is called a *normal magnetic curve* [**7**]. For extensive information about almost contact metric manifolds and Sasakian manifolds, we refer to Blair's book [**3**].

1.2. Quarter-symmetric metric connection.

Definition 1.3. A linear connection $\overline{\nabla}$ on an n-dimensional Riemannian manifold (M^n, g) is called a *quarter-symmetric connection* [8] if its torsion tensor T satisfies

$$T(X,Y) = \pi(Y)f(X) - \pi(X)f(Y),$$

where π is a $1-$form and f is a $(1,1)$-type tensor field. If, moreover, the connection $\overline{\nabla}$ satisfies

$$\left(\overline{\nabla}_X g\right)(Y,Z) = 0,$$

for all vector fields $X, Y, Z \in \chi(M)$, then it is called *quarter-symmetric metric connection.*

Let $\left(M^{2n+s}, \varphi, \xi_\alpha, \eta^\alpha, g\right)$ be an S-manifold. Let us take

$$\pi = -\sum_{\alpha=1}^{s} \eta_\alpha,\ \ f = \varphi.$$

Then, the affine metric connection defined by

$$\overline{\nabla}_X Y = \nabla_X Y + \sum_{\alpha=1}^{s} \eta_\alpha(X)\,\varphi Y \tag{5}$$

becomes a quarter-symmetric metric connection [**9**]. Here, ∇ denotes the Levi-Civita connection. It is noteworthy that Vanlı also used another quarter-symmetric metric connection in her recent paper [**20**], where she chose $\pi = \sum_{\alpha=1}^{s}\eta_\alpha,\ f = \varphi$. In our work, we use the connection given in (5), which will effect our equations only with a minus sign.

2. Quarter-symmetric Magnetic curves

2.1. Frenet curves.

Definition 2.1. Let (M^n, g) be an n-dimensional Riemannian manifold and $\gamma : I \to M$ a curve parametrized by arc-length. If there exists g-orthonormal vector fields $T = E_1, E_2, ..., E_r$ along γ such that

$$
\begin{aligned}
T &= E_1 = \gamma', \\
\overline{\nabla}_T T &= \overline{k}_1 E_2, \\
\overline{\nabla}_T E_2 &= -\overline{k}_1 T + \overline{k}_2 E_3, \\
&\ \ ... \\
\overline{\nabla}_T E_j &= -\overline{k}_{j-1} E_{j-1} + \overline{k}_j E_{j+1}, \ \ (2 < j < r), \\
&\ \ ... \\
\overline{\nabla}_T E_r &= -\overline{k}_{r-1} E_{r-1},
\end{aligned} \tag{6}
$$

then γ is called a *Frenet curve for* $\overline{\nabla}$ *of osculating order* r , $(1 \leq r \leq n)$ [14]. Here $\overline{k}_1, ..., \overline{k}_{r-1}$ are called *quarter-symmetric curvature functions of* γ and these functions are positive valued on I .

- A *geodesic for* $\overline{\nabla}$ (or *quarter-symmetric geodesic*) is a Frenet curve of osculating order 1 for $\overline{\nabla}$.
- If $r = 2$ and $\overline{k}_1$ is a constant, then γ is called a *quarter-symmetric circle.*
- A *quarter-symmetric helix of order* r, $r \geq 3$, is a Frenet curve for $\overline{\nabla}$ of osculating order r with non-zero positive constant quarter-symmetric curvatures $\overline{k}_1, ..., \overline{k}_{r-1}$.
- If we shortly state *quarter-symmetric helix,* we mean its osculating order is 3.

Let $M = \left(M^{2n+s}, \varphi, \xi_\alpha, \eta^\alpha, g\right)$ be an S-manifold endowed with the quarter-symmetric metric connection $\overline{\nabla}$. Let us denote the fundamental 2-form of M by Ω. Then, we have

$$\Omega(X, Y) = g(X, \varphi Y),$$

(see [**3**]). In an S-manifold, since $\Omega = d\eta^\alpha$ and $d^2 = 0$ (famous Poincaré identity), Ω is obviously closed. Thus, we can define a magnetic field F_q on M by

$$F_q(X, Y) = q\Omega(X, Y),$$

namely the *contact magnetic field with strength* q, where $X, Y \in \chi(M)$ and $q \in \mathbb{R}$ [**13**]. We will assume that $q \neq 0$ to avoid the absence of the strength of magnetic field (see [**5**] and [**7**]).

The Lorentz force Φ associated to the magnetic field F_q can be written as

$$\Phi = -q\varphi.$$

So the Lorentz equation (4) is

$$\nabla_T T = -q\varphi T, \tag{7}$$

where $\gamma : I \to M$ is a curve with arc-length parameter, $T = \gamma'$ is the tangent vector field and ∇ is the Levi-Civita connection (see **[7]** and **[13]**). By the use of equations (3) and (7), we have

$$\overline{\nabla}_T T = \left[-q + \sum_{\alpha=1}^{s} \eta_\alpha (T) \right] \varphi T. \tag{8}$$

Now, we can give the following definition:

Definition 2.2. Let $\gamma : I \to M$ be a unit-speed curve in an S-manifold $M = \left(M^{2n+s}, \varphi, \xi_\alpha, \eta^\alpha, g\right)$ endowed with the quarter-symmetric metric connection $\overline{\nabla}$. Then γ is called a *normal magnetic curve with respect to the quarter-symmetric metric connection* $\overline{\nabla}$ (or shortly *quarter-symmetric magnetic curve*) if it satisfies equation (8).

In **[9]**, it is shown that

$$\left(\overline{\nabla}_X \varphi\right)(Y) = \sum_{\alpha=1}^{s} \left\{ g(\varphi X, \varphi Y)\xi_\alpha + \eta^\alpha (Y)\varphi^2 X \right\},$$

$$\overline{\nabla}_X \xi_\alpha = -\varphi X, \ \alpha \in \{1, ..., s\}.$$

If we apply these equations along the unit-speed curve, we have

$$\begin{aligned} \left(\overline{\nabla}_T \varphi\right)(T) &= \sum_{\alpha=1}^{s} \left\{ g(\varphi T, \varphi T)\xi_\alpha + \eta^\alpha (T)\varphi^2 T \right\} \\ &= (1 - \sum_{\alpha=1}^{s} \cos^2 \theta_\alpha) \left(\sum_{\alpha=1}^{s} \xi_\alpha \right) \\ &\quad + \left(\sum_{\alpha=1}^{s} \cos \theta_\alpha \right) \left(-T + \sum_{\alpha=1}^{s} \cos \theta_\alpha \xi_\alpha \right) \end{aligned} \tag{9}$$

and

$$\overline{\nabla}_T \xi_\alpha = -\varphi T, \ \alpha \in \{1, ..., s\}.$$

Here, $\theta_\alpha = \theta_\alpha(t)$ denotes the angle functions between T and ξ_α, that is,

$$\cos \theta_\alpha(t) = g(T, \xi_\alpha).$$

Notice that γ is called a $\theta_\alpha-$slant curve if all θ_α are constants **[10]**. It is called a slant curve if these constant angles have the same value **[11]**. Moreover, if this common value is $\frac{\pi}{2}$ for all $\alpha = 1, 2, ..., s$, then it is called a Legendre curve **[18]**.

Now, we can give the following lemma:

Lemma 2.3. *A quarter-symmetric magnetic curve in an S-manifold is a* θ_α*-slant curve.*

Proof. Let $\gamma : I \to M$ be a quarter-symmetric magnetic curve. Then, we find

$$\begin{aligned}\frac{d}{dt}g(T,\xi_\alpha) &= g(\overline{\nabla}_T T,\xi_\alpha)+g(T,\overline{\nabla}_T\xi_\alpha)\\ &= g(\left[-q+\sum_{\alpha=1}^{s}\eta_\alpha(T)\right]\varphi T,\xi_\alpha)\\ &= 0.\end{aligned}$$

So, $\cos\theta_\alpha$ are constants for all $\alpha = 1, 2, ..., s$. □

Here is a direct corollary for $s = 1$:

Corollary 2.4. *In a Sasakian manifold endowed with $\overline{\nabla}$, a quarter-symmetric magnetic curve is slant.*

2.2. **Main Calculations.** Now, it is time to find quarter-symmetric curvatures of quarter-symmetric magnetic curves in S-manifolds.

Let $\gamma : I \to M = \left(M^{2n+s}, \varphi, \xi_\alpha, \eta^\alpha, g\right)$ be a quarter-symmetric magnetic curve. Since it is $\theta_\alpha-$slant, equation (8) becomes

$$\overline{\nabla}_T T = \left[-q+\sum_{\alpha=1}^{s}\cos\theta_\alpha\right]\varphi T. \tag{10}$$

Using (6) and (10), we have

$$\overline{k}_1 E_2 = \left[-q+\sum_{\alpha=1}^{s}\cos\theta_\alpha\right]\varphi T. \tag{11}$$

The norm of both sides in (11) gives us

$$\overline{k}_1 = \left|-q+\sum_{\alpha=1}^{s}\cos\theta_\alpha\right|\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}, \tag{12}$$

which is a constant. If we replace (12) in (11), we find

$$\varphi T = \delta\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}E_2, \tag{13}$$

where $\delta = sign\left(-q+\sum_{\alpha=1}^{s}\cos\theta_\alpha\right)$. If we differentiate both sides of (13) along the curve and use (9), we get

$$\begin{aligned}\overline{\nabla}_T\varphi T &= \left(1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha\right)\left(\sum_{\alpha=1}^{s}\xi_\alpha\right)\\ &\quad+\left(\sum_{\alpha=1}^{s}\cos\theta_\alpha\right)\left(-T+\sum_{\alpha=1}^{s}\cos\theta_\alpha\xi_\alpha\right)+\overline{k}_1\varphi E_2\\ &= \delta\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}\left(-\overline{k}_1 T+\overline{k}_2E_3\right).\end{aligned}$$

After some calculations, we have

$$\begin{aligned}\overline{k}_2\delta\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}E_3 &= \left(1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha\right)\left(\sum_{\alpha=1}^{s}\xi_\alpha\right)\\ &\quad+\left(\sum_{\alpha=1}^{s}\cos\theta_\alpha+\delta.D\right)\left(\sum_{\alpha=1}^{s}\cos\theta_\alpha\xi_\alpha\right)\\ &\quad-\left(\sum_{\alpha=1}^{s}\cos\theta_\alpha+\delta.D.\sum_{\alpha=1}^{s}\cos^2\theta_\alpha\right).T\\ &= W,\end{aligned}\tag{14}$$

where

$$D=\frac{\overline{\kappa}_1}{\sqrt{1-\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha}}=\left|-q+\sum_{\alpha=1}^{s}\cos\theta_\alpha\right|.$$

Finally, we obtain

$$\overline{k}_2=\sqrt{\begin{array}{c}\left(\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha\right)q^2-2\left(\sum\limits_{\alpha=1}^{s}\cos\theta_\alpha\right)\left(1+\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha\right)q\\ +\left(3+\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha\right)\left(\sum\limits_{\alpha=1}^{s}\cos\theta_\alpha\right)^2+\left(1-\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha\right)s\end{array}}\tag{15}$$

and

$$E_3=\frac{1}{\overline{k}_2\delta\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}}W.\tag{16}$$

Notice that $\overline{k}_2$ is also a constant.

If we differentiate (14) once more, we have $\overline{\kappa}_3=0$. As a result we can state our second lemma:

Lemma 2.5. *The osculating order of a quarter-symmetric magnetic curve in an S-manifold is at most* 3.

Finally, we can classify quarter-symmetric magnetic curves of S-manifolds as follows:

Theorem 2.6. *Let $\gamma : I \to M = \left(M^{2n+s}, \varphi, \xi_\alpha, \eta^\alpha, g\right)$ be a quarter-symmetric magnetic curve. Then it belongs to the following list:*

(a) quarter-symmetric non-Legendre $\theta_\alpha-$slant geodesics with $\sum_{\alpha=1}^{s}\cos\theta_\alpha \neq 0$ (including quarter-symmetric geodesics as integral curves of $\sum_{\alpha=1}^{s}\cos\theta_\alpha\xi_\alpha$, where $\sum_{\alpha=1}^{s}\cos^2\theta_\alpha = 1$); or

(b) quarter-symmetric $\theta_\alpha-$slant circles with

$$\overline{k}_1 = \left|-q + \sum_{\alpha=1}^{s}\cos\theta_\alpha\right| \sqrt{1 - \sum_{\alpha=1}^{s}\cos^2\theta_\alpha},$$

and its Frenet frame field for $\overline{\nabla}$ is

$$\left\{T, \frac{1}{\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}}\delta\varphi T\right\},$$

or

(c) quarter-symmetric $\theta_\alpha-$slant helices with

$$\overline{k}_1 = \left|-q + \sum_{\alpha=1}^{s}\cos\theta_\alpha\right| \sqrt{1 - \sum_{\alpha=1}^{s}\cos^2\theta_\alpha},$$

$$\overline{k}_2 = \sqrt{\begin{array}{c}\left(\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha\right)q^2 - 2\left(\sum\limits_{\alpha=1}^{s}\cos\theta_\alpha\right)\left(1+\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha\right)q \\ +\left(3+\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha\right)\left(\sum\limits_{\alpha=1}^{s}\cos\theta_\alpha\right)^2 + \left(1-\sum\limits_{\alpha=1}^{s}\cos^2\theta_\alpha\right)s\end{array}}$$

and its Frenet frame field for $\overline{\nabla}$ is

$$\left\{T, \frac{1}{\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}}\delta\varphi T, \frac{1}{\overline{k}_2\delta\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}}W\right\},$$

where W is given by (14).

Otherwise, γ can not be a quarter-symmetric magnetic curve.

Proof. Let $\gamma : I \to M$ be a quarter-symmetric magnetic curve of osculating order r. Then, γ is a $\theta_\alpha-$slant curve (Lemma 2.3) and $1 \leq r \leq 3$ (Lemma 2.5). Firstly, let $r = 1$, that is, $\overline{\nabla}_T T = 0$. From equation (10), we get

$$0 = \left[-q + \sum_{\alpha=1}^{s}\cos\theta_\alpha\right]\varphi T.$$

So, either $-q + \sum_{\alpha=1}^{s}\cos\theta_\alpha = 0$ or $\varphi T = 0$. If $-q + \sum_{\alpha=1}^{s}\cos\theta_\alpha = 0$, then γ is a non-Legendre $\theta_\alpha-$slant curve, since $q = \sum_{\alpha=1}^{s}\cos\theta_\alpha$ is a non-zero constant.

Or else, $\varphi T=0$ gives us $\varphi^2 T=0=-T+\sum_{\alpha=1}^{s}\cos\theta_\alpha\xi_\alpha$. Then, we can write

$$T=\sum_{\alpha=1}^{s}\cos\theta_\alpha\xi_\alpha,$$

i.e. γ is an integral curve of $\sum_{\alpha=1}^{s}\cos\theta_\alpha\xi_\alpha$. From the fact that γ is a unit-speed curve, we also have $g(T,T)=\sum_{\alpha=1}^{s}\cos^2\theta_\alpha=1$. We finished the proof of (a). Now, let $r=2$, that is, $\overline{k}_2=0$. In this case, the curve γ has only one non-zero quarter-symmetric curvature given by equation (12). If we denote the Frenet frame field as $\{T,E_2\}$, from equation (13), we can write

$$E_2=\frac{1}{\sqrt{1-\sum_{\alpha=1}^{s}\cos^2\theta_\alpha}}\delta\varphi T.$$

Note that $\overline{k}_1$ is a constant and $\overline{k}_2=0$, so γ is a quarter-symmetric $\theta_\alpha-$slant circle. The proof of (b) is done. Finally, let $r=3$, i.e. $\overline{k}_3=0$. We have already calculated the quarter-symmetric curvatures $\overline{k}_1$ and $\overline{k}_2$ in equations (12) and (15). If we denote the Frenet frame field as $\{T,E_2,E_3\}$, we can write E_2 and E_3 as in equations (13) and (16). Thus, the proof of (c) is over. The osculating order r can not be greater than 3, so the list is complete. If γ does not belong to the list, it can not be a quarter-symmetric magnetic curve. □

References

[1] Adachi ,T., Curvature bound and trajectories for magnetic fields on a Hadamard surface. Tsukuba J. Math. **20**, 225–230, (1996).

[2] Blair, D. E., Geometry of manifolds with structural group $U(n)\times O(s)$, J. Differential Geometry, **4**, 155-167, (1970).

[3] Blair, D. E., Riemannian geometry of contact and symplectic manifolds, Second edition. Progress in Mathematics, 203. Birkhauser Boston, Inc., Boston, MA, (2010).

[4] Barros M., Romero, A., Cabrerizo, J. L., Fernández, M., The Gauss-Landau-Hall problem on Riemannian surfaces. J. Math. Phys. **46**, no. 11, 112905, 15 pp, (2005).

[5] Cabrerizo, J. L., Fernandez M. and Gomez, J. S., On the existence of almost contact structure and the contact magnetic field. Acta Math. Hungar. **125**, 191-199, (2009).

[6] Comtet, A., On the Landau levels on the hyperbolic plane, Ann. Physics **173**, 185-209, (1987).

[7] Druţă-Romaniuc, S. L., Inoguchi, J., Munteanu, M. I., Nistor, A. I., Magnetic curves in Sasakian manifolds, Journal of Nonlinear Mathematical Physics, **22**, 428-447, (2015).

[8] Golab, S., On Semi-symmetric and quarter-symmetric connections, Tensor, N.S. **29**, 249-254, (1975).

[9] Göçmen, A., Quarter Symmetric Connections in S-manifolds, MSc Thesis, Supervisor: Prof. Dr. Aysel TURGUT VANLI, (2013).

[10] Güvenç, Ş., An Extended Family of Slant Curves in S-manifolds, Mathematical Sciences and Applications E-Notes 8 (1), 69-77, (2020).

[11] Güvenç, Ş., Özgür, C., On slant curves in S-manifolds, Communications of the Korean Mathematical Society 33 (1), 293-303, (2018).

[12] Güvenç, Ş., Özgür, C., On Pseudo-Hermitian Magnetic Curves in Sasakian Manifolds, Facta Universitatis, Series: Mathematics and Informatics 35 (5), 1291-1304, (2020).

[13] Jleli, M., Munteanu, M. I., Nistor, A. I., Magnetic trajectories in an almost contact metric manifold $\mathbb{R}^{2N+1}$, Results Math. **67**, 125-134, (2015).

[14] Jordan, C., , Sur la théorie des courbes dans l'espace à n dimensions, C. R. Acad. Sci. Paris 79, 795–797 (1874).
[15] Ledger, A., Espace de Riemann symetriques generalises, C. R. Acad. Sci. Paris 264, 947-948, (1967).
[16] Ledger, A., Obata, M., Affine and Riemannian s-manifolds, J. Differential Geometry 2, 451-459, (1968).
[17] Lee, J.-E., Pseudo-Hermitian magnetic curves in normal almost contact metric 3-manifolds, Communications of the Korean Mathematical Society, 35 (4), 1269-1281, (2020).
[18] Özgür, C., Güvenç, Ş., On Biharmonic Legendre Curves in S-Space Forms, Turkish Journal of Mathematics 38 (3), 454-461, (2014).
[19] Tsagas, Gr., Ledger, A., Riemannian S-manifolds, J. Differential Geometry 12, 333-343, (1977).
[20] Turgut Vanlı, A., Semi-Symmetry Properties of S-Manifolds Admitting a Quarter-Symmetric Metric Connection, Hagia Sophia Journal of Geometry, Vol. 2, No. 2, 38-47, 2020.
[21] Yano, K., Kon, M., Structures on Manifolds. Series in Pure Mathematics, **3**. Singapore. World Scientific Publishing Co. (1984).

Şaban Güvenç
Orcid number: 0000-0001-6254-4693
Department of Mathematics
Balikesir University
Balikesir, Turkey
Email address: sguvenc@balikesir.edu.tr