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THE RELATIONSHIP BETWEEN THE NUMBER OF EXTREMA OF COMPOUND SINUSOIDAL SIGNALS AND ITS HIGH-FREQUENCY COMPONENT

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Abstract. As the main findings of our research work, we present a novel theorem on the relationship between the number of extrema of compound sinusoidal signals and its high-frequency component. In the case of signals consisting of the sum of two sine signals, if the high-frequency component has a higher product of the frequency and the amplitude, then we prove that the frequency of the high-frequency component is proportional to the number of extrema in a time interval. This theorem justifies some of the experimental results of other researchers on the relevance of extrema to frequency and amplitude. To confirm the theorem, extrema counting was performed on speech signals and compared with Fourier transform. The experimental results show that the average number of extrema of the compound sinusoidal signal or its derivatives over a time interval can be used to estimate the frequency at its highest frequency band. An important application of this research work is the fast calculation of high frequencies of a signal. This theorem also shows that the number of extrema points can be used as a new effective feature for signal processing, especially speech signals.

Keywords: Extreme points, Composite wave, Turning points, Time-frequency analysis, Spectral estimation, Empirical mode decomposition
2020 MSC: 68Txx, 68T10

1. Introduction

Computational complexity and extracting powerful features for signal processing, especially speech signals, is a fundamental problem. In most applications, frequency-domain features are used for signal processing, but transforming to the frequency domain is time-consuming. In this work, we are looking for the relationship between the number of extrema of a signal and some of its frequency features in order to extract frequency features in the time domain to reduce computational complexity.

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1.1. **History.** In recent decades, some methods have been proposed to approximate the instantaneous frequency features of a signal in the time domain [9, 12, 14, 22, 26]. The main aim of these studies is to reduce the computational complexity caused by transferring to the frequency domain or to extract a particular type of frequency feature in the time domain. Although fast Fourier transform (FFT) [6] is able to accurately compute the frequency components of a signal, since many signals, such as speech signals, are non-stationary, this type of transformation is inefficient. Thus, the methods such as short-time Fourier transform (STFT) and wavelet transform have been proposed [2, 8, 28]. Wavelet transform has various applications in time-varying signal processing [1, 3, 28, 29]. Also, by Chen et al., statistical methods in the frequency domain have been used to estimate the instantaneous frequency accurately [7]. However, most time-domain methods have less computational complexity than the frequency domain, and the results are intuitive and easier to analyze. So, in this article, we are looking for time-domain methods.

For the relationship between time and frequency domains, the Hilbert-Huang transform (HHT) was proposed by Huang et al. in 1998 [13]. HHT is a method for analyzing nonlinear and non-stationary signals based on Empirical Mode Decomposition (EMD) and Hilbert transform. Since the signal is decomposed in the time domain and the length of the IMFs (Intrinsic Mode Functions-IMF) is the same as the original signal, HHT preserves the characteristics of the varying frequency. The EMD method was also first introduced in 1998 by Huang et al. The EMD is the main part of HHT, based on the extrema and zero-crossing points [13]. The EMD decomposes the signal by direct extraction of local energy associated with the intrinsic time scales of the signal at sequences of local extrema. Thus, the method is similar to our work and traditional decompositions such as Fourier or Wavelet. Therefore, EMD can be interpreted as a type of wavelet decomposition with sub-bands produced as needed during the extraction process of the IMFs, which represents the details of the signals on a desired scale or frequency range [26].

The EMD method has many applications: EMD is used for frequency decomposition of the signal [9, 22], audio coding [5], speech enhancement [20], extracting spectral information from voice signals and analysis [12], signal feature extraction [16], random noise suppression [17] and filtering [4]. EMD has different versions: Ensemble EMD (EEMD) [14, 27, 30], improved EMD based on local integral mean [19], EMD and Hurst-based (EMDH) [20], improved EMD using second-generation wavelets interpolation [29] and wavelet-based empirical mode decomposition [1]. In addition, Extrema analysis has many applications in signal decomposition.

Extrema analysis is used to sample, encode, transform, and reconstruct the time-varying signals: Joy et al. proposed a simple transform based on extrema points of the signal. The transformed value at a given point is calculated based on the distance and magnitude difference of two extrema points it lies between, rather than considering every point around it [14]. Premanand and

Sheeba sampled and encoded high-frequency signals using extremum sampling [21]. Meignen and Gumery proposed a novel approach to the reconstruction of finite signal derivatives from the extrema of a multiscale representation, first introduced by Berkner. They also focused on signal approximation from the multiscale extrema representation of one of its derivatives [18].

Extrema analysis is used for speech segmentation and clustering: Ghosh, by measuring the distance between consecutive extrema (extrema-based signal track length-ESTL) as a time-domain feature, has segmented the speech signals and shown that the distance between consecutive extrema is proportional to the product of the instantaneous frequency and amplitude of a sinusoidal signal. It has also compared ESTL-based segmentation with ML (Maximum Likelihood) and STM (Spectral Transition Measure) methods and found that it is as good as spectral feature-based segmentation, but with a lesser computational complexity [10]. Gokcesu proposed a nonparametric extrema analysis that can be used in envelope extraction, peak-burst detection, and clustering in time series [11].

Extrema analysis is used to estimate the instantaneous amplitude, frequency, temporal bandwidth, and feature extraction of time-varying signals: Sharma and Sreenivas, only by using extrema samples and a local polynomial regression based least-squares fitting approach, attempted to estimate the instantaneous amplitude and frequency in a time-varying sinusoid [25]. Rzepka and Miskowicz showed that the temporal bandwidth of time-varying signals can be dynamically estimated by sampling the signal at its local extrema and reconstructing the signal. They also showed that for the bandlimited Gaussian random processes, the average rate of extremum sampling is directly proportional to the maximum frequency component in the signal spectrum [23]. Seifpour et al. proposed a time-domain feature named Statistical Behavior of Local Extrema (SBLE) for automatic sleep staging [24]. In this article, we present and prove a theorem that shows the relevance of the number of extrema with the high-frequency component of a signal in a time interval. This theorem confirms some of the experimental results of other researchers regarding the relation of extrema with frequency and amplitude [4, 10, 13, 14, 18, 21, 23, 25].

1.2. **Preliminaries.** Extremum is defined as any point at which the local value of a function is the largest (a maximum) or the smallest (a minimum). Extrema of a continuously differentiable function can be found by examining the zero-crossing of the first derivative of that function [15]. Extrema and zero-crossing points are time-domain features that approximately indicate the periodic aspect of the signal, so in some applications of EMD, these features have been used to filter high-frequency signal noises [4]. According to the proposed theorem, the average number of extrema over an interval can be expressed as an estimate of the frequency in a high-frequency band. To clarify the subject, consider a simple sinusoidal signal with the equation $x(t) = A_0 sin(2\pi f_0 t + \varphi_0)$, where A_0 is the amplitude, f_0 is the frequency, and φ_0 is the phase. This signal oscillates f_0 times per second, two extrema exist in each oscillation, and in total, there are $2f_0$ extrema per second. Therefore, if we divide the number of extrema over an interval by twice the length of the interval, the frequency of the signal is obtained. Suppose that $z(t)$ is the sum of $x(t) = sin(2\pi 100t)$ and $y(t) = sin(2\pi 350t)$ as shown in Figure 1. We are looking for the relationship between the number of extrema of $z(t)$ with the frequency of its components. It is empirically observed that the number of extrema of $z(t)$ and $y(t)$ is identical. Generally speaking, the number of $z(t)$ extrema per second is twice the frequency of the signal $y(t)$. What is mentioned above will be considered further in the proposed theorem.

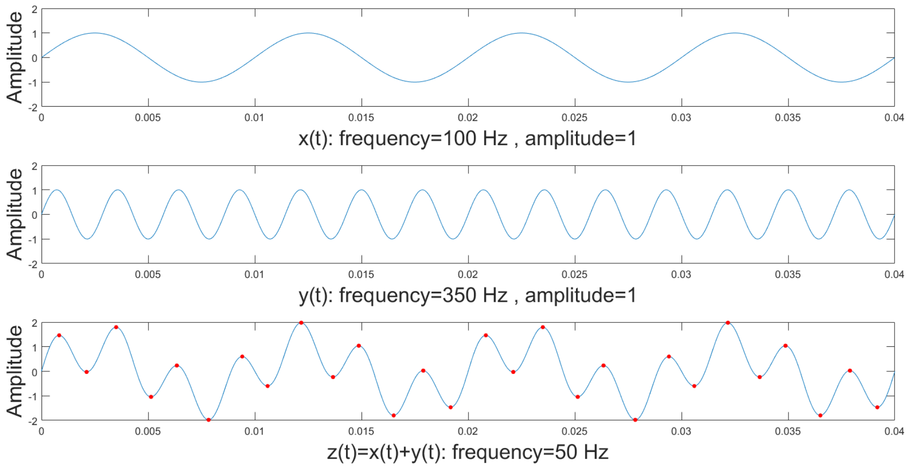

FIGURE 1. The sum of two signals with the same amplitude and different frequencies. Note the relationship between the number of extrema and the frequency of its components. Extrema are marked in red.

Given that the human auditory system works in the frequency domain but the computer systems sample the audio signals in the time domain, it is essential to consider the relationship between the frequency-domain features and the time-domain features. In this paper, two related theorems on the relationship between the number of extrema with the high-frequency component of sine signals are presented and proved. The first theorem states a particular case of the second one. By applying these theorems directly in the time domain and using the number of extrema, the frequency of the high-frequency component of some types of signals can be computed.

This paper has been organized as follows: Section 2 describes the theorems, lemmas, and their proof. Numerical validation and discussion are presented in

Section 3. Finally, the conclusion and suggestions for possible future research are presented.

2. The theorem of the relationship between the number of extrema of sinusoidal signals and the high-frequency component

In this section, for the first time, a theorem (Theorem 2.4) about the relationship between the number of extremum points and the frequency of components of a signal is presented and proved, which confirms the experimental findings of researchers in this field and opens a way for further research. First, to clarify the problem, Theorem 2.1 is presented regarding the relationship between extrema and the frequency of compound sinusoidal signals with same-amplitude components, which is a special case of Theorem 2.4, so only Theorem 2.4 is proved. To prove Theorem 2.4, the first two lemmas are presented and proved.

Theorem 2.1. *The number of extrema in a period of a signal consisting of two sinusoidal signals of the same amplitude with different frequencies is twice that of the component frequency, which has the higher frequency.*

Theorem 2.1 expresses that, in the case of signals with the same amplitude, the number of extrema is related to the components of the signal that have a higher frequency. In the case where two signals with different amplitudes are combined, we present Theorem 2.4 as "the relationship between the number of extrema of sinusoidal signals and high-frequency component." Since Theorem 2.4 is a generalization of Theorem 2.1, we only prove Theorem 2.4. Let us first prove the following two lemmas that are needed to prove Theorem 2.4:

Any cosine wave can be represented as $a(t) = A_0 cos(2\pi f_0 t + \varphi_0)$, with the parameters A_0 being the amplitude, f_0 being the frequency, and φ_0 being the phase. Lemma 2.2 states that in a sinusoidal signal composed of two cosine waves, the wave that has a lower frequency and amplitude has a lower slope at points with the same magnitude.

Lemma 2.2. *Consider two arbitrary cosine waves $x(t)$ and $y(t)$, according to Equations*

$$x(t) = A_1 cos(2\pi f_1 t + \varphi_1), \tag{1}$$

$$y(t) = A_2 cos(2\pi f_2 t + \varphi_2), \tag{2}$$

and conditions

$$|f_1| < |f_2|, \tag{3}$$

$$|A_1| < |A_2|. \tag{4}$$

In these conditions, the absolute value of the slope of the curve $x(t)$ at an arbitrary time t_1 is less than the absolute value of the slope of the curve $y(t)$ at *the time t_2 with the same magnitude. In other words:*

$$x(t_1) = y(t_2) \Rightarrow |x'(t_1)| < |y'(t_2)|. \tag{5}$$

Proof. Suppose that the conditions of Lemma 2.2 hold; to prove Lemma 2.2, we start from the left-hand side of Equ. 5 and show that the right-hand side is true (see Figure 2 for a better understanding):

$$\begin{aligned} &x(t_1) = y(t_2) \Rightarrow |x(t_1)| = |y(t_2)| \Rightarrow \\ &|A_1 cos(2\pi f_1 t_1 + \varphi_1)| = |A_2 cos(2\pi f_2 t_2 + \varphi_2)| \Rightarrow \\ &|cos(2\pi f_1 t_1 + \varphi_1)| = \frac{|A_2|}{|A_1|}|cos(2\pi f_2 t_2 + \varphi_2)|. \end{aligned} \tag{6}$$

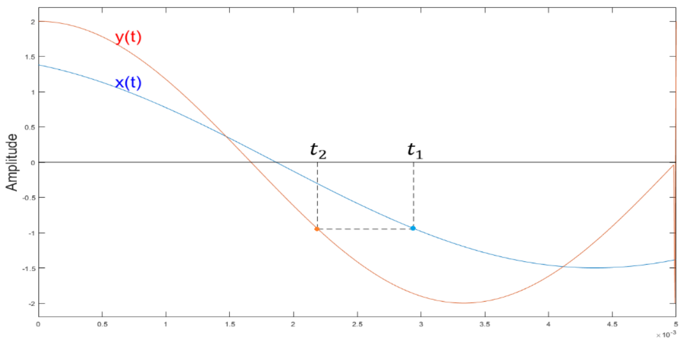

FIGURE 2. The slope of the two signals at points of the same magnitude. According to Lemma 2.2, the absolute value of the slope of $x(t)$ at t_1 is less than the absolute value of the slope of $y(t)$ at t_2.

From Equs. 4 and 6 we will have:

$$|cos(2\pi f_1 t_1 + \varphi_1)| > |cos(2\pi f_2 t_2 + \varphi_2)|, \tag{7}$$

when the cosine of one angle is greater than the cosine of another angle, its sine will be smaller than the sine of the other angle, so from Equ. 7, then we have:

$$|sin(2\pi f_1 t_1 + \varphi_1)| < |sin(2\pi f_2 t_2 + \varphi_2)|, \tag{8}$$

from Conditions 3 and 4, we have $|-2\pi A_1 f_1| < |-2\pi A_2 f_2|$, so from Equ. 8:

$$|-2\pi A_1 f_1 sin(2\pi f_1 t_1 + \varphi_1)| < |-2\pi A_2 f_2 sin(2\pi f_2 t_2 + \varphi_2)|,$$

the derivative of $x(t)$ and $y(t)$ are:

$$x(t) = -2\pi A_1 f_1 sin(2\pi f_1 t_1 + \varphi_1), \quad y(t) = -2\pi A_2 f_2 sin(2\pi f_2 t_2 + \varphi_2),$$

then we have:

$$|x'(t_1)| < |y'(t_2)|.$$

and the proof of Lemma 2.2 ends. □

Lemma 2.3. *Let the sinusoids $x(t)$ and $y(t)$ be according to Equs. 1 and 2 and Conditions 3 and 4, then $x(t)$ intersects $y(t)$ in a period of $y(t)$ from a maximum to its next maximum at exactly two points.*

Proof. Consider Figure 3. In this figure, the frequency and amplitude of $y(t)$ are higher than the frequency and amplitude of $x(t)$, respectively. We want to prove that $x(t)$ intersects $y(t)$ in the interval $[t1, t3]$ only at two points (the opposite is not correct). To achieve this, we divide $[t1, t3]$ into two subintervals $[t1, t2]$ (from a maximum $y(t)$ to the next minimum) and $[t2, t3]$ (from the minimum $y(t)$ to the next maximum); then we prove that in each interval, $x(t)$ intersects $y(t)$ only at one point. First, we prove this in the interval $[t1, t2]$.

Consider the following argument: Since $|A_1| < |A_2|$, then the maximum point of $x(t)$ is less than the maximum point of $y(t)$ and the minimum point of $x(t)$ is greater than the minimum point of $y(t)$, meaning that all values of $x(t)$ are between the maximum and the minimum $y(t)$; $y(t)$ is continuous and strictly descending from its maximum to minimum point in $[t1, t2]$; $x(t)$ is continuous; at the beginning of the interval $[t1, t2]$, the $x(t)$ is less than $y(t)$, and at the end of it, $x(t)$ is greater than $y(t)$. Therefore, $x(t)$ necessarily crosses between the maximum and minimum of $y(t)$ and intersects it at least at one point.

Now we prove that it cannot intersect any more than one point: In the interval $[t1, t2]$, $x(t)$ is less than $y(t)$ before intersecting; therefore, after the intersection, $x(t)$ is greater than $y(t)$, in order that $x(t)$ to intersect $y(t)$ once again in $[t1, t2]$, the absolute value of the slope of $x(t)$ must be greater than the absolute value of the slope of $y(t)$ at the next intersection, but according to Lemma 2.2, this is not possible. Also, these two signals can never be tangent because at the tangent point, the slopes of the two signals are equal, and it is not possible according to Lemma 2.2. Therefore, $x(t)$ intersects $y(t)$ in the interval $[t1, t2]$ only at one point. In the same manner, and concerning symmetry, this can be proved in the interval $[t2, t3]$; thus, the proof of Lemma 2.3 is completed. Now we prove the main theorem. □

Theorem 2.4. *If $z(t)$ is composed of the sum of two sinusoidal signals, and the higher frequency component has a higher product of the frequency and the amplitude, then the number of extrema of $z(t)$ in a period will be equal to the number of extrema of the higher frequency component in the same period. In other words, if $z(t)$ is composed of the sum of two sine signals $x(t)$ and $y(t)$ as*

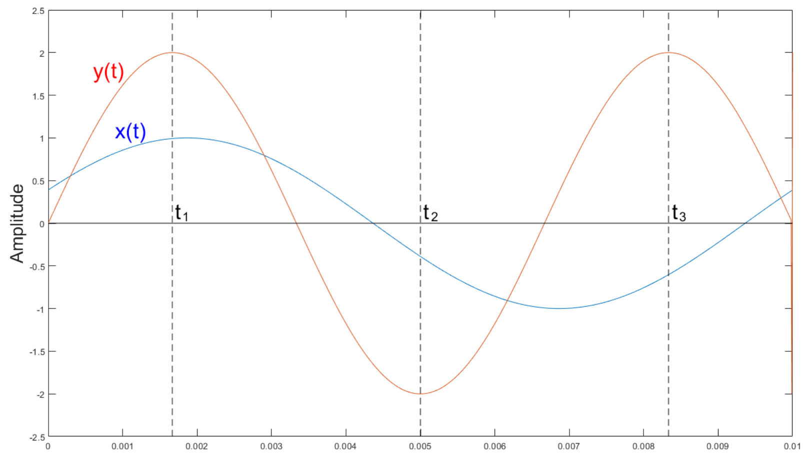

FIGURE 3. Number of intersections of two sinusoidal signals in a period of the higher frequency signal.

follows:

$$z(t) = x(t) + y(t),$$
$$x(t) = A_1 sin(2\pi f_1 t + \varphi_1),$$
$$y(t) = A_2 sin(2\pi f_2 t + \varphi_2),$$

and Conditions

(9) $$|f_2| > |f_1|, \frac{f_1}{f_2} \in Q,$$

(10) $$|A_2 f_2| > |A_1 f_1|.$$

In these circumstances, the number of extrema of $z(t)$ in one of its periods is equal to the number of extrema of $y(t)$ in the same period. Q is the set of rational numbers.

Proof. First, we prove that $z(t)$ is periodic and compute its period: Let $T_1 = 1/|f_1|$ and $T_2 = 1/|f_2|$ are the smallest period of the functions $x(t)$ and $y(t)$, respectively. Since f_1/f_2 is rational, then T_1/T_2 will also be rational, and can be represented as follows:

$$\frac{T_1}{T_2} = \frac{n}{m}.$$

where n and m are positive integers and coprime to each other. Since the value of mT_1 (or nT_2) is divisible by both T_1 and T_2, then it is the common period of both $x(t)$ and $y(t)$. Therefore $z(t)$, which is the sum of these two signals, is also periodic and its period is $T = mT_1$. Note that the period of a function is not necessarily a rational number. Now we find the number of extrema of the function $z(t)$ in a period. Since $z(t)$ is continuous and differentiable, then the points at which $z'(t)$ (the derivative of $z(t)$) changes sign, or in other words, intersect the time axis, will be extrema:

$$z'(t) = 2\pi A_1 f_1 cos(2\pi f_1 t + \varphi_1) + 2\pi A_2 f_2 cos(2\pi f_2 t + \varphi_2),$$

So, the number of roots of $z'(t)$ in the period T is equal to the number of extrema of $z(t)$. By placing $z'(t) = 0$, we will have:

$$2\pi A_1 f_1 cos(2\pi f_1 t + \varphi_1) + 2\pi A_2 f_2 cos(2\pi f_2 t + \varphi_2) = 0,$$

then:

$$A_1 f_1 cos(2\pi f_1 t + \varphi_1) = -A_2 f_2 cos(2\pi f_2 t + \varphi_2). \tag{11}$$

There is no analytical solution to find the roots of Equ. 11, but the number of roots can be found in a given interval. We denote the left side of Equ. 11 with $x_1(t)$ and the right side of it with $y_1(t)$:

$$x_1(t) = A_1 f_1 cos(2\pi f_1 t + \varphi_1), \tag{12}$$

$$y_1(t) = -A_2 f_2 cos(2\pi f_2 t + \varphi_2), \tag{13}$$

To find the number of roots of Equ. 11 in a period of $z(t)$, it is enough to find the number of intersection points of $x_1(t)$ and $y_1(t)$ in the period T. From Condition 10, it is concluded that the absolute value of the amplitude of $y_1(t)$ is greater than the absolute value of the amplitude of $x_1(t)$. According to Lemma 2.3 and Conditions 9, 10 and Equs. 12 and 13, it is concluded that the number of the intersection points of $x_1(t)$ and $y_1(t)$ in a period of $y_1(t)$ from a maximum to its next maximum is exactly two points. On the other, $y_1(t)$ intersects the time axis twice in its period. Therefore, the number of intersection points of the $x_1(t)$ and $y_1(t)$ (the number of roots of Equ. 11) is equal to the number of roots of $y_1(t)$ in any period of it; since T (period of $z(t)$) is the integer multiple of the period $y_1(t)$, the same is true at the period T. On the other, the number of roots of $y_1(t)$ in a given interval is equal to the number of extrema of the integral $y_1(t)$:

$$\int y_1(t)dt = \frac{(-A_2)}{2\pi} sin(2\pi f_2 t + \varphi_2). \tag{14}$$

The number of extrema of Equ. 14 is equal to the number of extrema of the function $y(t)$ at each distance. Therefore, the number of roots of Equ. 11 or, in other words, the number of extrema of $z(t)$ is equal to the number of extrema of $y(t)$ at the period T, and the proof of Theorem 2.4 is completed. In the case of $A1 = A2$, Theorem 2.4 becomes Theorem 2.1, so Theorem 2.1 does not need to be proved. □

3. Numerical validation and discussion

As mentioned, there are two extrema in each period of a simple sinusoidal signal; therefore, if the second condition of Theorem 2.4 (Condition 10) is satisfied, by using the number of extrema and through Equ. 15, we can compute the frequency of the component having the highest frequency:

$$f = \frac{m}{2T}. \tag{15}$$

In Equ. 15, T is the period of the main signal $z(t)$, and m is the number of extrema in the interval T. If the period of $z(t)$ is unknown, the value of T can be replaced by a greater value, and the extrema are counted in that interval to compute the frequency with relatively good accuracy.

Theorem 2.4 has two limitations. The first limitation is the second condition of Theorem 2.4, which may not be satisfied for many signals. This condition emphasizes a component of the signal that has the desired amplitude in addition to having the highest frequency, so this can be applied to known signals that achieve this condition. The second limitation is that Theorem 2.4 cannot be generalized to the sum of more than two sinusoidal signals. These limitations make Theorem 2.4 not applicable for many signals, especially speech signals which have a noisy nature, but by using Theorem 2.4 and some considerations, high-frequency components can be approximated. In speech signal processing, it is better to remove the noise with a suitable filter so that the speech is not corrupted and then use the results of this theorem.

As there are two extrema in each period of a sinusoidal signal, there are also two turning points, so in Theorem 2.4, the number of turning points can be used instead of the number of extrema. If the conditions of Theorem 2.4 are not met, it is more appropriate to use the number of turning points to compute the high-frequency component. In the absence of these conditions, some extrema of the high-frequency component with relatively low amplitude may be converted into turning points, so counting the turning points will be more accurate. In this case, the experimental results also showed that the number of extrema of the second derivative of the main signal is more accurate than the number of turning points. Likewise, the higher the derivative, the better the results, but the computational accuracy will be extremely decreased. To evaluate the above, the following arbitrary signal, consisting of two sinusoidal signals, was chosen:

$$z(t) = 4sin(2\pi 300t) + A_2 sin(2\pi 500t + 1),$$

To make the number of extrema proportional to the $500Hz$ component frequency, the second condition of Theorem 2.4 requires that:

$$A_2 * 500 > 4 * 300 \Rightarrow A_2 > 2.4. \tag{16}$$

The experimental results showed that to compute the 500 Hz component frequency by the number of extrema, Condition 16 must be maintained; but, when using the number of turning points or extrema of the second derivative, the value of 2.4 can be reduced to 0.97 and 0.87 respectively (Table 1). Also, the experimental results on different signals showed that in the absence of the conditions of Theorem 2.4 to compute the frequency, the use of the number of turning points or the number of second derivative extrema yields more accurate results than using the number of extrema of the main signal.

Among the advantages of using the proposed theorem are high calculation speed, a simple algorithm, and the need for a lower sampling rate, and as a result, it requires less memory. As a disadvantage, the condition in the presented theorem is not valid for many signals, but still, extrema can be used as an effective feature.

Table 1. Counting the number of sign changes of the first to third derivatives and computing the frequency from it. The value inserted in the third column is the minimum required amplitude of the high-frequency component (A_2), which contradicts the second condition of Theorem 2.4; however, the frequency computation is still performed without mistake.

Method		Minimum allowed values of A_2
Number of extrema of the main signal	Changing the sign of the first derivative	2.4
Number of extrema of the first derivative (turning points)	Changing the sign of the second derivative	0.97
Number of extrema of the second derivative	Changing the sign of the third derivative	0.87

4. **Conclusion and future research**

The human auditory system is sensitive to the two parameters of the amplitude and frequency of sound so that the higher the amplitude or energy of a sound, the better it will be heard; likewise, the higher the frequency of a signal to a defined boundary, the better it will be heard. Therefore, the components of a speech signal, whose product of the amplitude and the frequency is greater than the others, are heard better. Also, according to Theorem 2.4, the number of extrema corresponds to a component that product of the amplitude, and the frequency is greater than the others. Thus, by using extrema, it is possible to find the component of the speech signal to which the auditory system is more sensitive. In other words, the auditory system is sensitive to the number of extrema in each time interval of the speech signal.

The component mentioned in the previous paragraph can also be found using the fast Fourier transform; it should be noted that the time to compute

FFT is of order n $log_2 n$, but the time to count extrema is of order n, which is much faster. On the other hand, extrema are in the time domain, which represent the intuitive features of the signal. Theorem 2.4 shows that there is a relationship between the extrema and the frequency of the components of a signal so that the number of extrema can be used as an effective feature in signal processing, especially speech signals. According to Theorem 2.4, this feature is useful in cases such as dealing with high-frequency components of the signal. The extraction of this feature, unlike the fast Fourier transform, has low time complexity and a simple algorithm.

The main contribution is to present a new theorem related to sinusoidal signals and to prove it. Also, we found an application of it to calculate the frequency of components of the signal. According to the applications discussed in this article and comparing our method with conventional methods, the following advantages and disadvantages can be mentioned:

Advantages:

(1) Low computational complexity
(2) Simple algorithm
(3) Low sampling rate and thus less memory and processing
(4) A new, intuitive, and effective feature for speech processing
(5) Based on the human auditory system

Disadvantages:

(1) Not applicable to some signals.
(2) It is not valid for all frequency components of the signal.

Therefore, the proposed method has no general application, so it can be used to process signals that approximately provide the conditions of the theorem. Finding more applications of the theorem is one of our future tasks. Other researchers are requested to find more applications for this theorem. There may be a simpler proof for this theorem, which is left to other researchers. Also, further investigation of the extrema of higher derivatives and presentation and proof of related theorems are among the future works.

5. Author Contributions

The contributions of the authors are as follow: Conceptualization, M. Nikzar and H. khotanoul; methodology, M. Nikzar and H. Khotanlou; software, M.Nikzar; validation, M. Nikzar, H. Khotanlou; investigation, M. Nikzar, H.Khotanlou and M. Dezfoulian; writing-original draft preparation, M. Nikzar; writing-review and editing, H. Khotanlou and M. Dezfoulian; visualization, M. Nikzar; supervision, H. Khotanlou and M. Dezfoulian; project administration, H. Khotanlou. All authors have read and agreed to the published version of the manuscript.

6. Data Availability Statement

Not applicable.

7. Ethical considerations

Not applicable.

8. Funding

This research received no funding.

9. Conflict of interest

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analysis, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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