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DOMESTIC VIOLENCE AND HOMICIDE: FUZZY SIMILARITY MEASURES

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Abstract. In this paper, we determine the fuzzy similarity measures of the U. S. state rankings with respect to domestic violence, female homicide, and sexual violence against teens. We find that the fuzzy similarity measures are low. We then consider the best state rankings and determine the fuzzy similarity measures of this ranking a with the previous three rankings. We also develop some theoretical results concerning fuzzy similarity measures.

Keywords: Domestic violence, female homicide, sexual violence against teenagers, state rankings, fuzzy similarity measures.

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1. Introduction

The following is taken from [1]. Domestic violence is a pervasive issue in the United States. It affects people of all genders, sexualities, ethnicities, and backgrounds. In America, domestic violence can range from physical assault to emotional assault. Its effects on survivors can include depression, post-traumatic stress disorder, anxiety, and other mental health issues. According to data from the National Coalition Against Domestic Violence, about 20 people per minute experience physical violence at the hands of an intimate partner - that's more than 10 million Americans every year. Moreover, about 1 in 4 women and 1 in 9 men experience severe physical violence by an intimate partner at some point during their lives. This violent behavior impacts victims directly but also has serious repercussions for families and communities as a whole: those affected may struggle to maintain employment or housing; children exposed to domestic violence are more likely to suffer from behavioral problems; and communities with high rates of such abuse often experience higher-than-average levels of crime. Unfortunately, due to its secretive nature violence remains largely unrecognized and untreated across the nation. We apply similarity measures to rankings of members of a finite set.

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For a set X, we let $\mathcal{FP}(X)$ denote the fuzzy power set of X, i.e., the set of all fuzzy subset of X. We let $\wedge$ denote the minimum (or infimum) of a set of real numbers and the $\vee$ the maximum (or supremum). For two fuzzy subsets μ, ν of X, we write $\mu \subseteq \nu$ if $\mu(x) \leq \nu(x)$ for all $x \epsilon X$.

Suppose that X is a set with n elements, i.e., $|X| = n$.Let A be a one-to-one function of X onto $\{1, 2, ..., n\}$.Then A is called a **ranking** of X. Define the

fuzzy subset μ_A of X into $[0, 1]$ by for all $x \epsilon X$, $\mu_A(x) = A(x)/n$.Then μ_A is called the **fuzzy subset associated** with A. Let A be a ranking of a set X. Suppose X has n elements. Define $A^* : X \to \{1, 2, ..., n\}$ as follows: $\forall x \epsilon X$,

$$A^*(x) = n + 1 - A(x).$$

Then A^* is a ranking of X. Also,

$$\mu_{A^*}(x) = 1 + \frac{1}{n} - \mu_A(x).$$

It should be noted that A^* is the ranking of X that yields the smallest fuzzy similarity measure A can have with any ranking of X, [4]. Note also that for all $x \epsilon X$,

$$\mu_{A^*}(x) = \mu_{A^c}(x) + \frac{1}{n},$$

where μ_{A^c} is the complement of μ_A. We call A^* the **reverse ranking** of A.

Let A be a ranking of X. Then since A is a one-to-one function of X onto $\{1, 2, ..., n\}$,

$$\begin{aligned}
\sum_{x \in X} A(x) &= 1 + 2 + \cdots + n = \frac{n(n+1)}{2}, \\
\sum_{x \in X} \mu_A(x) &= \sum_{x \in X} \frac{A(x)}{n} = \frac{1}{n} \sum_{x \in X} A(x) = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2}.
\end{aligned}$$

Definition 1.1. Let S be a function of $\mathcal{FP}(X) \times \mathcal{FP}(X)$ into $[0, 1]$. Then S is called a **fuzzy similarity measure** on $\mathcal{FP}(X)$ if the following properties hold: $\forall \mu, \nu, \rho \epsilon \mathcal{FP}(X)$

(1) $S(\mu, \nu) = S(\nu, \mu)$;
(2) $S(\mu, \nu) = 1$ if and only if $\mu = \nu$;
(3) If $\mu \subseteq \nu \subseteq \rho$, then $S(\mu, \rho) \leq S(\mu, \nu) \wedge S(\nu, \rho)$;
(4) If $S(\mu, \nu) = 0$, then $\forall x \epsilon X$, $\mu(x) \wedge \nu(x) = 0$.

Example 1.2. *Let μ_A and μ_B be fuzzy subsets associated with two rankings A and B of X, respectively. Then M and S are fuzzy similarity measures, where*

$$\begin{aligned}
M(\mu_A, \mu_B) &= \frac{\sum_{x \epsilon X} \mu_A(x) \wedge \mu_B(x)}{\sum_{x \epsilon} \mu_A(x) \vee \mu_B(x)}, \\
S(\mu_A, \mu_B) &= 1 - \frac{\sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|}{\sum_{x \epsilon} (\mu_A(x) + \mu_B(x))}.
\end{aligned}$$

Theorem 1.3. *[3] Let μ_A and μ_B be fuzzy subsets associated with two rankings A and B of X, respectively. Let M and S be defined as in Example 1.2. Then $S = \frac{2M}{M}$ and $M = \frac{S}{2-S}$.*

Theorem 1.4. *[4] Let X be a finite set with n elements. Let μ_A and μ_B be fuzzy subsets of X associated with two rankings A and B of X, respectively.*

(1) If n is even, then the smallest $M(\mu_A, \mu_B)$ can be is $\frac{n+2}{3n+2}$ and the smallest $S(\mu_A, \mu_B)$ is $\frac{n/2+1}{n+1}$.

(2) If n is odd, then the smallest $M(\mu_A, \mu_B)$ can be is $\frac{n+1}{3n-1}$ and the smallest $S(\mu_A, \mu_B)$ is $\frac{1}{2} + \frac{1}{2n}$.

Consider Theorem 1.4. Suppose that s denotes the smallest value for S. Define

$$\widehat{S}(\mu_A, \mu_B) = \frac{S(\mu_A, \mu_B) - s}{1 - s}.$$

Then $\widehat{S}(\mu_A, \mu_B)$ varies between 0 and 1. For values of $\widehat{S}(\mu_A, \mu_B)$ between 0 and 0.2, we say the the fuzzy similarity is very low, between 0.2 and 0.4 low, between 0.4 and 0.6 medium, between 0.6 and 0.8 high, and between 0.8 and 1 very high. A similar approach can be taken for M.

The following proposition follows immediately.

Proposition 1.5. $\widehat{S}(\mu_A, \mu_B) - \widehat{S}(\mu_{A^*}, \mu_B) = \frac{S(\mu_A, \mu_B) - S(\mu^*, \mu_B)}{1-s}$, *where s is the smallest element S can be.*

2. Domestic Violence, Female Homicide, and Sexual Violence Against Teens

In the following table, we provide the rankings of states with respect to Domestic Violence (DV), Female Homicide (FH), and Sexual Violence Against Teens (SVAT). The rankings were determined from [1]. (See Tables 7-9, Appendix A.) We then find the fuzzy similarity measures of the rankings.

We first delete the countries that are not present in both rankings and then rerank.

Domestic Violence vs Female Homicide

Here $n = 48$.

$$S = 1 - \frac{570}{2322} = 1 - 0.2455 = 0.7545.$$

The smallest S can be is

$$\text{Smallest } S = \frac{n/2 + 1}{n + 1} = \frac{25}{49} = 0.5102.$$

Thus

$$\widehat{S} = \frac{0.7545 - 0.5102}{1 - 0.5102} = 0.4988.$$

TABLE 1. Violence and Homicide

State	DV vs FH	DV vs SVAT	FH vs SVAT
Alabama		15 / 20	
Alaska	3 / 1	3 / 29	1 / 27
Arizona	4 / 11	4 / 31	7 / 29
Arkansas	10 / 9	7 / 32	8 / 30
California	33 / 28	23 / 34	20 / 32
Colorado	26 / 12		
Connecticut	20 / 41		
Delaware	21 / 38	13 / 1	26 / 1
Florida		12 / 14	
Georgia	23 / 31	16 / 13	21 / 13
Hawaii	34 / 48	24 / 6	33 / 6
Idaho	44 / 32	32 / 35	22 / 33
Illinois	8 / 44	6 / 19	30 / 18
Indiana	5 / 30		
Iowa	31 / 47	21 / 7	32 / 7
Kansas	37 / 16	27 / 15	11 / 14
Kentucky	1 / 25	1 / 2	16 / 2
Louisiana	29 / 5		
Maine	17 / 34		
Maryland	36 / 24	26 / 5	17 / 5
Massachusetts	38 / 46	28 / 4	31 / 4
Michigan	28 / 15	19 / 30	10 / 28
Minnesota	39 / 45		
Mississippi	14 / 18	10 / 11	13 / 11
Missouri	7 / 7		

Now

$$M = \frac{S}{2-S} = \frac{0.7545}{2-0.7545} = 0.6058.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5102}{2-0.5102} = 0.3425.$$

Thus

$$\widehat{M} = \frac{0.6058-0.3425}{1-0.3425} = 0.4005.$$

The fuzzy similarity measure is medium.

Domestic Violence vs Sexual Violence Against Teens

Here $n = 35$.

$$S = 1 - \frac{405}{1260} = 1 - 0.3214 = 0.6786.$$

Table 2. Violence and Homicide (continued)

State	DV vs FH	DV vs SVAT	FH vs SVAT
Montana	24 / 29	17 / 24	19 / 22
Nebraska	41 / 40	29 / 25	28 / 23
Nevada	2 / 3	2 / 8	3 / 8
New Hampshire	35 / 4	25 / 21	4 / 19
New Jersey	30 / 42	20 / 28	29 / 26
New Mexico	22 / 2	14 / 18	2 / 17
New York	46 / 35	33 / 12	24 / 12
North Carolina	32 / 27	22 / 22	18 / 20
North Dakota	47 / 37	34 / 9	25 / 9
Ohio	19 / 6		
Oklahoma	11 / 8	8 / 23	6 / 21
Oregon	13 / 23		
Pennsylvania	25 / 39	18 / 27	27 / 25
Rhode Island	45 / 26		
South Carolina	6 / 6	5 / 17	5 / 16
South Dakota	48 / 19	35 / 16	14 / 15
Tennessee	15 / 10		
Texas	12 / 20	9 / 10	15 / 10
Utah	42 / 33	30 / 33	23 / 31
Vermont	18 / 13		
Virginia	43 / 14	31 / 3	9 / 3
Washington	9 / 22		
West Virginia	16 / 17	11 / 26	12 / 24
Wisconsin	27 / 21		
Wyoming	40 / 43		

The smallest S can be is

$$\text{Smallest } S = \frac{1}{2} + \frac{1}{2(35)} = 0.5143.$$

Thus

$$\widehat{S} = \frac{0.6786 - 0.5143}{1 - 0.5143} = 0.3383.$$

Now

$$M = \frac{S}{2-S} = \frac{0.6786}{2 - 0.6786} = 0.5136.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5143}{2 - 0.5143} = 0.346\,2.$$

Thus

$$\widehat{M} = \frac{0.5136 - 0.3462}{1 - 0.3462} = 0.2560.$$

The fuzzy similarity measure is low.

Female Homicide vs Sexual Violence Against Teens

Here $n = 33$.

$$S = 1 - \frac{402}{1122} = 1 - 0.3583 = 0.6417.$$

The smallest S can be is

$$\text{Smallest } S = \frac{1}{2} + \frac{1}{2(33)} = 0.5152.$$

Thus

$$\widehat{S} = \frac{0.6417 - 0.5152}{1 - 0.5152} = 0.260\,9.$$

Now

$$M = \frac{S}{2 - S} = \frac{0.6417}{2 - 0.6417} = 0.4724.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5152}{2 - 0.5152} = 0.3470.$$

Thus

$$\widehat{M} = \frac{0.4724 - 0.3470}{1 - 0.3470} = 0.1920.$$

The fuzzy similarity measure is low.

3. Best States for a Woman to Live

In [6], rankings of states in U.S. with respect to the best place for women were provided. It is stated in [1] that their comprehensive measure of women's well-being, rights, and opportunities in the United states reveals vast differences across the United States. the Index is structured around three basic dimensions: inclusion (economic, social, political), justice (formal laws and informal discrimination), and security (at the individual discrimination levels). The index and its 12 indicators, grouped into these three dimensions, provide a standardized, quantitative, and transparent measuring for the ranking of all states. We provide the ranking in the following table and then we find the fuzzy similarity measures of this with domestic violence, female homicide, and sexual violence against teens.

Domestic Violence vs Best State

Here $n = 50$.

$$S = 1 - \frac{1042}{2550} = 1 - 0.4086 = 0.5914.$$

The smallest S can be is

$$\text{Smallest } S = \frac{n/2 + 1}{n + 1} = \frac{26}{51} = 0.5098.$$

Thus

$$\widehat{S} = \frac{0.5914 - 0.5098}{1 - 0.5098} = 0.1665.$$

TABLE 3. Best State to Live vs Violence

State	BS vs DV	BS vs FH	BS vs SVAT
Alabama	47 / 24		33 / 20
Alaska	27 / 3	27 / 1	17 / 29
Arizona	30 / 4	29 / 11	20 / 31
Arkansas	48 / 10	46 / 9	34 / 32
California	14 / 35	14 / 28	8 / 34
Colorado	13 / 28	13 / 12	
Connecticut	2 / 21	2 / 41	
Delaware	21 / 22	21 / 38	13 / 1
Florida	29 / 20		19 / 14
Georgia	36 / 25	35 / 31	25 / 13
Hawaii	9 / 36	9 / 48	5 / 6
Idaho	38 / 46	37 / 32	26 / 35
Illinois	12 / 8	12 / 44	7 / 19
Indiana	33 / 5	32 / 30	
Iowa	22 / 33	22 / 47	14 / 7
Kansas	25 / 39	25 / 16	15 / 15
Kentucky	46 / 1	45 / 25	32 / 2
Louisiana	50 / 31	48 / 5	
Maine	8 / 17	8 / 34	
Maryland	6 / 38	6 / 24	3 / 5
Massachusetts	1 / 40	1 / 46	1 / 4
Michigan	20 / 30	20 / 15	12 / 30
Minnesota	11 / 41	11 / 45	
Mississippi	49 / 14	47 / 18	35 / 11
Missouri	37 / 7	36 / 7	
Montana	31 / 26	30 / 29	21 / 24

Now

$$M = \frac{S}{2-S} = \frac{0.5914}{2-0.5914} = 0.4199.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5098}{2-0.5098} = 0.3421.$$

Thus

$$\widehat{M} = \frac{0.4199-0.3421}{1-0.3421} = 0.1183.$$

The fuzzy similarity measure is very low.

Female Homicide vs Best State

Here $n = 48$.

$$S = 1 - \frac{940}{2352} = 1 - 0.3997 = 0.6003.$$

TABLE 4. Best State to Live vs Violence (Continued)

State	BS vs DV	BS vs FH	BS vs SVAT
Nebraska	18 / 43	18 / 40	10 / 25
Nevada	34 / 2	33 / 3	23 / 8
New Hampshire	5 / 37	5 / 4	2 / 21
New Jersey	10 / 32	10 / 42	6 / 28
New Mexico	39 / 23	38 / 2	27 / 18
New York	7 / 48	7 / 35	4 / 12
North Carolina	32 / 34	31 / 27	22 / 22
North Dakota	19 / 49	19 / 37	11 / 9
Ohio	24 / 19	24 / 36	
Oklahoma	41 / 11	40 / 8	29 / 23
Oregon	17 / 13	17 / 23	
Pennsylvania	16 / 27	16 / 39	9 / 27
Rhode Island	4 / 47	4 / 26	
South Carolina	43 / 6	42 / 6	30 / 17
South Dakota	28 / 50	28 / 19	18 / 16
Tennessee	44 / 15	43 / 10	
Texas	40 / 12	39 / 20	28 / 10
Utah	35 / 44	34 / 33	24 / 33
Vermont	3 / 18	3 / 13	
Virginia	26 / 45	26 / 14	16 / 3
Washington	23 / 9	23 / 22	
West Virginia	45 / 16	44 / 17	31 / 26
Wisconsin	15 / 29	15 / 21	
Wyoming	42 / 42	41 / 43	

The smallest S can be is

$$\text{Smallest } S = \frac{n/2+1}{n+1} = \frac{25}{49} = 0.5102.$$

Thus

$$\widehat{S} = \frac{0.6003 - 0.5102}{1 - 0.5102} = 0.1840.$$

Now

$$M = \frac{S}{2-S} = \frac{0.6003}{2 - 0.6003} = 0.4289.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5102}{2 - 0.5102} = 0.3425.$$

Thus

$$\widehat{M} = \frac{0.4289 - 0.3425}{1 - 0.3425} = 0.1314.$$

The fuzzy similarity measure is very low.

Sexual Violence Against Teens vs Best State

Here $n = 35$.

$$S = 1 - \frac{376}{1260} = 1 - 0.2984 = 0.7016.$$

The smallest S can be is

$$\text{Smallest } S = \frac{1}{2} + \frac{1}{2(35)} = 0.5143.$$

Thus

$$\widehat{S} = \frac{0.7016 - 0.5143}{1 - 0.5143} = 0.3856.$$

Now

$$M = \frac{S}{2-S} = \frac{0.7016}{2 - 0.7016} = 0.5404.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5143}{2 - 0.5143} = 0.3462.$$

Thus

$$\widehat{M} = \frac{0.5404 - 0.3462}{1 - 0.3462} = 0.2970.$$

The fuzzy similarity measure is low.

The low fuzzy similarity measures found above are to be expected. A small number in the ranking for Best State (BS) means the state is high with respect to a place to live, while a small number for DV, FH, and SVAT means the state is high with respect to these negative issues. In the following table, we consider the fuzzy similarity measure for the reverse ranking BS* of BS.

Domestic Violence vs Best State*

Here $n = 50$.

$$S = 1 - \frac{642}{2550} = 1 - 0.2518 = 0.7482.$$

The smallest S can be is

$$\text{Smallest } S = \frac{n/2 + 1}{n + 1} = \frac{26}{51} = 0.5098.$$

Thus

$$\widehat{S} = \frac{0.7482 - 0.5098}{1 - 0.5098} = 0.4863.$$

Now

$$M = \frac{S}{2-S} = \frac{0.7482}{2 - 0.7482} = 0.5977.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5098}{2 - 0.5098} = 0.3421.$$

Thus

$$\widehat{M} = \frac{0.5977 - 0.3421}{1 - 0.3421} = 0.3885.$$

The fuzzy similarity measure is low.

Table 5. Best States vs Violence and Homicide

State	BS* / DV	BS* / FH	BS* / SVAT
Alabama	4 / 24		3 / 20
Alaska	24 / 3	22 / 1	19 / 29
Arizona	21 / 4	20 / 11	16 / 31
Arkansas	3 / 10	3 / 9	2 / 32
California	37 / 35	35 / 28	28 / 34
Colorado	38 / 28	36 / 12	
Connecticut	49 / 21	47 / 41	
Delaware	30 / 22	28 / 38	23 / 1
Florida	22 / 20		17 / 14
Georgia	15 / 25	14 / 31	11 / 13
Hawaii	42 / 36	40 / 48	31 / 6
Idaho	13 / 46	12 / 32	10 / 35
Illinois	39 / 8	37 / 44	29 / 19
Indiana	18 / 5	17 / 30	
Iowa	29 / 33	27 / 47	22 / 7
Kansas	26 / 39	24 / 16	21 / 15
Kentucky	5 / 1	4 / 25	4 / 2
Louisiana	1 / 31	1 / 5	
Maine	43 / 17	41 / 34	
Maryland	45 / 38	43 / 24	33 / 5
Massachusetts	50 / 40	48 / 46	35 / 4
Michigan	31 / 30	29 / 15	24 / 30
Minnesota	40/41	38 / 45	
Mississippi	2 / 14	2 / 18	1 / 11
Missouri	14 / 7	13 / 7	
Montana	20 / 26	19 / 29	15 / 24

We have $\widehat{S}(DV, BS^*) - \widehat{S}(DV, BS) = 0.4863 - 0.1665 = 0.3198$. Thus the effect of DV on is positive, but low.

Female Homicide vs Best State*

Here $n = 48$.

$$S = 1 - \frac{566}{2352} = 1 - 0.2406 = 0.7594.$$

The smallest S can be is

$$\text{Smallest } S = \frac{n/2 + 1}{n + 1} = \frac{25}{49} = 0.5102.$$

Thus

$$\widehat{S} = \frac{0.7594 - 0.5102}{1 - 0.5102} = 0.5088.$$

Table 6. Best States vs Violence and Homicide (Continued)

State	BS* / DV	BS* / FH	BS* / SVAT
textNebraska	33 / 43	31 / 40	26 / 25
Nevada	17 / 2	16 / 3	13 / 8
New Hampshire	46 / 37	44 / 4	34 / 21
New Jersey	41 / 32	39 / 42	30 / 28
New Mexico	12 / 23	11 / 2	9 / 18
New York	44 / 48	42 / 35	32 / 12
North Carolina	19 / 34	18 / 27	14 / 22
North Dakota	32 / 49	30 / 37	25 / 9
Ohio	27 / 19	25 / 36	
Oklahoma	10 / 11	9 / 8	7 / 23
Oregon	34 / 13	32 / 23	
Pennsylvania	35 / 27	33 / 39	27 / 27
Rhode Island	47 / 47	45 / 26	
South Carolina	8 / 6	7 / 6	6 / 17
South Dakota	23 / 50	21 / 19	18 / 16
Tennessee	7 / 15	6 / 10	
Texas	11 / 12	10 / 20	8 / 10
Utah	16 / 44	15 / 33	12 / 33
Vermont	48 / 18	46 / 13	
Virginia	25 / 45	23 / 14	20 / 3
Washington	28 / 9	26 / 22	
West Virginia	6 / 16	5 / 17	5 / 26
Wisconsin	36 / 29	34 / 21	
Wyoming	9 / 42	8 / 43	

Now

$$M = \frac{S}{2-S} = \frac{0.7594}{2-0.7594} = 0.6121.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5102}{2-0.5102} = 0.3425.$$

Thus

$$\widehat{M} = \frac{0.6121-0.3425}{1-0.3425} = 0.4100.$$

The fuzzy similarity measure is medium.

We have $\widehat{S}(FH, BS^*) - \widehat{S}(FH, BS) = 0.5088 - 0.1840 = 0.3248$. This suggests that FH has a low effect on BS.

Sexual Violence Against Teens vs Best State*

Here $n = 35$.

$$S = 1 - \frac{436}{1260} = 1 - 0.3460 = 0.6540.$$

The smallest S can be is

$$\text{Smallest } S = \frac{1}{2} + \frac{1}{2(35)} = 0.5143.$$

Thus

$$\widehat{S} = \frac{0.6540 - 0.5143}{1 - 0.5143} = 0.2876.$$

Now

$$M = \frac{S}{2 - S} = \frac{0.6540}{2 - 0.6540} = 0.4859.$$

The smallest M can be is

$$\text{Smallest } M = \frac{0.5143}{2 - 0.5143} = 0.3462.$$

Thus

$$\widehat{M} = \frac{0.4859 - 0.3462}{1 - 0.3462} = 0.2137.$$

The fuzzy similarity measure is low.

We have $\widehat{S}(SVAT, BS^*) - \widehat{S}(SVAT, BS) = 0.2876 - 0.3856 = -0.0980$. This suggests that $SVAT$ has no effect on BS.

Other papers dealing with violence are [2] and [5].

4. **Theory**

In this section, we develop theoretical results to compare the fuzzy similarity measure of two rankings with the fuzzy similarity measure of one of the rankings and the reverse ranking of the other.

Let A and B be rankings of X. Throughout, we assume B satisfies, for all $x \epsilon X$, either (1) or (2) holds, where
(1) $\mu_A(x) \leq \mu_B(x) \leq \mu_{A^*}(x)$ and
(2) $\mu_A(x) \geq \mu_B(x) \geq \mu_{A^*}(x)$.
Let

$$E = \{x \epsilon X \mid A(x) = B(x) = A^*(x)\}$$

and

$$X_i = \{x \epsilon X \mid x \notin E \text{ and } (i) \text{ holds}\},$$

$i = 1, 2$. Let n_i denote the cardinality of X_i, $i = 1, 2$.

Note that

$$E \cap X_1 = \emptyset,\ E \cap X_2 = \emptyset,\ X_1 \cap X_2 = \emptyset.$$

Now

$$S(\mu_A, \mu_B) = 1 - \frac{\sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|}{\sum_{x \epsilon X} (\mu_A(x) + \mu_B(x))}$$

and

$$S(\mu_{A^*}, \mu_B) = 1 - \frac{\sum_{x \epsilon X} |\mu_{^*A}(x) - \mu_B(x)|}{\sum_{x \epsilon X} (\mu_{A^*}(x) + \mu_B(x))}$$

Theorem 4.1. *Suppose that* A, A^*, *and* B *satisfy* (1) *and* (2). *Then*

$$S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) = \frac{1}{n+1} \left(n_1 - n_2 + \frac{n_1}{n} - \frac{n_2}{n} - \sum_{x \epsilon X_1} 2\mu_B(x) + \sum_{x \epsilon X_2} 2\mu_B(x) \right).$$

Proof. Now

$$\begin{aligned}
S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) &= 1 - \frac{\sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|}{\sum_{x \epsilon X} (\mu_A(x) + \mu_B(x))} - \left(1 - \frac{\sum_{x \epsilon X} |\mu_{^*A}(x) - \mu_B(x)|}{\sum_{x \epsilon X} (\mu_{A^*}(x) + \mu_B(x))} \right) \\
&= \frac{\sum_{x \epsilon X} |\mu_{^*A}(x) - \mu_B(x)|}{\sum_{x \epsilon X} (\mu_{A^*}(x) + \mu_B(x))} - \frac{\sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|}{\sum_{x \epsilon X} (\mu_A(x) + \mu_B(x))} \\
&= \frac{\sum_{x \epsilon X} |\mu_{^*A}(x) - \mu_B(x)|}{\sum_{x \epsilon X} \mu_{A^*}(x) + \sum_{x \epsilon X} \mu_B(x)} - \frac{\sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|}{\sum_{x \epsilon X} \mu_A(x) + \sum_{x \epsilon X} \mu_B(x)} \\
&= \frac{\sum_{x \epsilon X} |\mu_{^*A}(x) - \mu_B(x)|}{\frac{n+1}{2} + \frac{n+1}{2}} - \frac{\sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|}{\frac{n+1}{2} + \frac{n+1}{2}} \\
&= \frac{\sum_{x \epsilon X} |\mu_{^*A}(x) - \mu_B(x)|}{n+1} - \frac{\sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|}{n+1} \\
&= \frac{\sum_{x \epsilon X} |\mu_{^*A}(x) - \mu_B(x)| - \sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|}{n+1}
\end{aligned}$$

Next we evaluate, $\sum_{x \epsilon X} |\mu_{^*A}(x) - \mu_B(x)| - \sum_{x \epsilon X} |\mu_A(x) - \mu_B(x)|$.

We have that

$$\begin{aligned}
&\sum_{x\epsilon X}|\mu_{A^*}(x)-\mu_B(x)|-\sum_{x\epsilon X}|\mu_A(x)-\mu_B(x)|\\
=\ &\sum_{x\epsilon E}|\mu_{A^*}(x)-\mu_B(x)|+\sum_{x\epsilon X_1}|\mu_{A^*}(x)-\mu_B(x)|+\sum_{x\epsilon X_2}|\mu_{A^*}(x)-\mu_B(x)|\\
&-\sum_{x\epsilon E}|\mu_A(x)-\mu_B(x)|-\sum_{x\epsilon X_1}|\mu_A(x)-\mu_B(x)|-\sum_{x\epsilon X_2}|\mu_A(x)-\mu_B(x)|\\
=\ &0+\sum_{x\epsilon X_1}(\mu_{A^*}(x)-\mu_B(x))+\sum_{x\epsilon X_2}(\mu_B(x)-\mu_{A^*}(x))\\
&-0-\sum_{x\epsilon X_1}(\mu_B(x)-\mu_A(x))-\sum_{x\epsilon X_2}(\mu_A(x)-\mu_B(x))\\
=\ &\sum_{x\epsilon X_1}(\mu_{A^*}(x)-\mu_B(x))+\sum_{x\epsilon X_2}(\mu_B(x)-\mu_{A^*}(x))-\sum_{x\epsilon X_1}(\mu_B(x)-\mu_A(x))-\sum_{x\epsilon X_2}(\mu_A(x)-\mu_B(x))\\
=\ &\sum_{x\epsilon X_1}(\mu_{A^*}(x)+\mu_A(x))-\sum_{x\epsilon X_2}(\mu_{A^*}(x)+\mu_A(x))-2\sum_{x\epsilon X_1}\mu_B(x)+2\sum_{x\epsilon X_2}\mu_B(x)\\
=\ &\sum_{x\epsilon X_1}(1+\frac{1}{n}-\mu_A(x)+\mu_A(x))-\sum_{x\epsilon X_2}(1+\frac{1}{n}-\mu_A(x)+\mu_A(x))-2\sum_{x\epsilon X_1}\mu_B(x)+2\sum_{x\epsilon X_2}\mu_B(x)\\
=\ &\sum_{x\epsilon X_1}(1+\frac{1}{n})-\sum_{x\epsilon X_2}(1+\frac{1}{n})-2\sum_{x\epsilon X_1}\mu_B(x)+2\sum_{x\epsilon X_2}\mu_B(x)\\
=\ &n_1(1+\frac{1}{n})-n_2(1+\frac{1}{n})-2\sum_{x\epsilon X_1}\mu_B(x)+2\sum_{x\epsilon X_2}\mu_B(x)\\
=\ &(n_1-n_2)(1+\frac{1}{n})-2\sum_{x\epsilon X_1}\mu_B(x)+2\sum_{x\epsilon X_2}\mu_B(x).
\end{aligned}$$

Hence,

$$\begin{aligned}
S(\mu_A,\mu_B)-S(\mu_{A^*},\mu_B) &= \frac{\sum_{x\epsilon X}|\mu_{^*A}(x)-\mu_B(x)|-\sum_{x\epsilon X}|\mu_A(x)-\mu_B(x)|}{n+1}\\
&= \frac{(n_1-n_2)(1+\frac{1}{n})-2\sum_{x\epsilon X_1}\mu_B(x)+2\sum_{x\epsilon X_2}\mu_B(x).}{n+1}\\
&= \frac{1}{n+1}\left((n_1-n_2)(1+\frac{1}{n})-2\sum_{x\epsilon X_1}\mu_B(x)+2\sum_{x\epsilon X_2}\mu_B(x)\right).
\end{aligned}$$

□

Theorem 4.2. *Let $|X|=n$ and A, A^*, B are rankings of X satisfying* (1) *and* (2). *Then the following assertions hold.*

(i) $E\neq\emptyset$ if and only if n is odd. Moreover, if $E\neq\emptyset$, then $|E|=1$.

(ii) Let $x\in X$. Then $x\in X_1$ if and only if there exists $y\in X$ such that $y\in X_2$.

(iii) $|X_1| = |X_2|$.

(iv) Let $X = \{x_1, x_2, \ldots, x_n\}$ *be such that* $A(x_i) = i$ *for all* $i = 1, 2, \ldots, n$. *Then* $B(x_i) = A(x_i) = i$ *or* $B(x_i) = A^*(x_i) = n + 1 - i$ *for all* $i = 1, 2, \ldots, n$. *Moreover, if* $B(x_i) = A(x_i) = i$, *then* $B(x_{n+1-i}) = A^*(x_i) = n + 1 - i$, *and if* $B(x_i) = A(x_i) = n+1-i$, *then* $B(x_{n+1-i}) = A^*(x_i) = i$ *for all* $i = 1, 2, \ldots, n$.

Proof. (i) Suppose $E \neq \emptyset$. Let $x \in E$. Then

$$\begin{aligned} \mu_A(x) &= \mu_B(x) = \mu_{A^*}(x) \\ \mu_A(x) &= \mu_{A^*}(x) \\ \frac{A(x)}{n} &= \frac{A^*(x)}{n} \\ A(x) &= A^*(x) \\ A(x) &= (n+1) - A(x) \\ 2A(x) &= n+1 \\ n &= 2A(x) - 1. \end{aligned}$$

This implies that n is odd.

Conversely suppose n is odd. Since n is odd, $n + 1$ is even and so $\frac{n+1}{2} \in \{1, 2, \ldots, n\}$. Since A is onto $\{1, 2, \ldots, n\}$, there exists $x \in X$ such that

$$A(x) = \frac{n+1}{2}.$$

Then

$$A^*(x) = (n+1) - A(x) = (n+1) - \frac{n+1}{2} = \frac{n+1}{2} = A(x).$$

This implies that

$$\mu_A(x) = \mu_{A^*}(x).$$

Since (1) and (2) hold, it follows that

$$\mu_A(x) = \mu_B(x) = \mu_{A^*}(x).$$

Thus, $x \in E$. Hence, $E \neq \emptyset$.

Next we show that $|E| = 1$. Since $x \in E$, $|E| \geq 1$.

Suppose $u \in X$ and $u \in E$. Then

$$\begin{aligned} \mu_A(u) &= \mu_B(u) = \mu_{A^*}(u) \\ \mu_A(u) &= \mu_{A^*}(u) \\ A(u) &= A^*(u) \\ A(u) &= (n+1) - A(u) \\ 2A(u) &= n+1 \\ A(u) &= \frac{n+1}{2} = A(x). \end{aligned}$$

Since A is one-to-one, $A(u) = A(x)$ implies that $u = x$. It now follows that $E = \{x\}$ and so $|E| = 1$.

(ii) Suppose $x \in X_1$. Let us write $A(x) = a$. Since $x \notin E$, either $\mu_A(x) \neq \mu_B(x)$ or $\mu_{A^*}(x) \neq \mu_B(x)$. Suppose $\mu_A(x) \neq \mu_B(x)$. Then

$$\begin{aligned} \mu_A(x) &< \mu_B(x) \leq \mu_{A^*}(x) \\ \mu_A(x) &< \mu_{A^*}(x) \\ A(x) &< A^*(x) = n + 1 - A(x) \\ a &< (n+1) - a. \end{aligned}$$

Now $b = (n+1) - a \in \{1, 2, \ldots, n\}$ and A is onto $\{1, 2, \ldots, n\}$. Thus there exists $y \in X$ such that $A(y) = b$. Now,

$$A^*(y) = (n+1) - A(y) = (n+1) - b = (n+1) - ((n+1) - a) = a.$$

Hence

$$A(y) = b = (n+1) - a > a = A^*(y).$$

Since $A(y) > A^*(y)$ and (1) and (2) are satisfied, it follows that

$$A(y) \geq B(y) > A^*(y) \text{ or } A(y) > B(y) \geq A^*(y).$$

This implies that $y \in X_2$. Similarly, if $\mu_{A^*}(x) \neq \mu_B(x)$, then there exists $y \in X$ such that $y \in X_2$.

Conversely, in a similar manner, we can show that if $y \in X_2$, then there exists $x \in X$ such that $x \in X_1$.

(iii) This follows from (ii).

(iv) First we suppose n is even. Then $n = 2m$ for some integer m. Thus, $X = \{x_1, x_2, \ldots, x_{m-1}, x_m, x_{m+1}, x_{m+2}, \ldots, x_{2m}\}$.

First we consider $B(x_m)$. Now,

$$A(x_m) = m \text{ and } A^*(x_m) = 2m + 1 - m = m + 1.$$

Since (1) and (2) holds and $m < m + 1$, it follows that

$$m = A(x_m) \leq B(x_m) \leq A^*(x_m) = m + 1.$$

Also,

$$m + 1 = A(x_{m+1}) \geq B(x_{m+1}) \geq A^*(x_{m+1}) = 2m + 1 - (m + 1) = m.$$

It now follows that $B(x_m) = m$ or $m + 1$ and $B(x_{m+1}) = m$ or $m + 1$. Since B is one-to-one, if $B(x_m) = m$, then $B(x_{m+1}) = m + 1$ and if $B(x_m) = m + 1$, then $B(x_{m+1}) = m$. That is, if $B(x_m) = A(x_m)$, then $B(x_{m+1}) = A^*(x_{m+1})$ and if $B(x_m) = A^*(x_{m+1})$, then $B(x_{m+1}) = A(x_m)$.

It also follows that $m, m + 1 \in \{1, 2, \ldots, m - 1, m, m + 1, m + 2, \ldots, 2m\}$ have been assigned preimages in X.

Suppose that if $B(x_j) = A(x_j)$, then $B(x_{2m+1-j}) = A^*(x_{2m+1-j})$ and if $B(x_j) = A^*(x_{2m+1-j})$, then $B(x_{2m+1-j}) = A(x_j)$ for $j = i + 1, \ldots, m - 1, m$.

We now consider $B(x_i)$, where $i < m$. Now $A(x_i) = i$ and $A^*(x_i) = 2m+1-i$. It is easy to see that $i < 2m + 1 - i$, i.e., $A(x_i) < A^*(x_i)$ and so since (1) and (2) hold

$$A(x_i) \leq B(x_i) \leq A^*(x_i), \text{ i.e., } i \leq B(x_i) \leq 2m + 1 - i.$$

Now by our assumption, $i+1, \ldots, m-1, m, m+1, \ldots, 2m-i \in I_n$ have been assigned their preimages. Hence, it now follows that

$$B(x_i) = i \text{ or } B(x_i) = 2m+1-i. \qquad (*)$$

We have

$$A(x_{2m+1-i}) = 2m+1-i, \;\; A^*(x_{2m+1-i}) = 2m+1-(2m+1-i) = i \text{ and } 2m+1-i > i.$$

Thus,

$$A(x_{2m+1-i}) \geq A^*(x_{2m+1-i}).$$

Since (1) and (2) holds, it follows that

$$\begin{aligned} A(x_{2m+1-i}) &\geq B(x_{2m+1-i}) \geq A^*(x_{2m+1-i}), \\ \text{i.e., } 2m+1-i &\geq B(x_{2m+1-i}) \geq i. \end{aligned}$$

Since B is one-to-one and $i+1, \ldots, m-1, m, m+1, \ldots, 2m-i \in I_n$ have been assigned their preimages, it follows that

$$B(x_{2m+1-i}) = i \text{ or } B(x_{2m+1-i}) = 2m+1-i. \qquad (**)$$

From (*), (**), and since B is one-to-one, it follows that if $B(x_i) = i = A(x_i)$, then $B(x_{2m+1-i}) = 2m+1-i = A^*(x_{2m+1-i})$, and if $B(x_i) = 2m+1-i = A^*(x_i)$, then $B(x_{2m+1-i}) = i = A^*(x_{2m+1-i})$.

We can continue this process assign value to B as follows: if $B(x_i) = A(x_i) = i$, then $B(x_{n+1-i}) = A^*(x_i) = n+1-i$, and if $B(x_i) = A(x_i) = n+1-i$, then $B(x_{n+1-i}) = A^*(x_i) = i$ for all $i = 1, 2, \ldots, n$.

Now suppose n is odd. Then $n = 2m+1$ for some integer m. Consider $B(x_{m+1})$. Now $A(x_{m+1}) = m+1$ and $A^*(x_{m+1}) = n+1-(m+1) = 2m+1+1-(m+1) = m+1$. Thus, $A(x_{m+1}) = A^*(x_{m+1})$. Since (1) and (2) hold, it follows that

$$A(x_{m+1}) = B(x_{m+1}) = A^*(x_{m+1}) = m+1.$$

For $i = 1, 2, \ldots, n = 2m+1$, $i \neq m+1$, value to $B(x_i)$ is assigned as in the case when n is even.

□

Corollary 4.3. *Suppose that* A, A^*, *and* B *satisfy* (1) *and* (2). *Then*

$$S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) = \frac{1}{n+1}\left(2\sum_{x \epsilon X_2} \mu_B(x) - 2\sum_{x \epsilon X_1} \mu_B(x)\right).$$

Proof. By Theorem 4.2 (iii), $|X_1| = |X_2|$, i.e., $n_1 = n_2$. Hence by Theorem 4.1,

$$\begin{aligned} S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) &= \frac{1}{n+1}\left((n_1 - n_2)(1 + \frac{1}{n}) - 2\sum_{x\epsilon X_1} \mu_B(x) + 2\sum_{x\epsilon X_2} \mu_B(x)\right) \\ &= \frac{1}{n+1}\left((n_1 - n_1)(1 + \frac{1}{n}) - 2\sum_{x\epsilon X_1} \mu_B(x) + 2\sum_{x\epsilon X_2} \mu_B(x)\right) \\ &= \frac{1}{n+1}\left(0 \cdot (1 + \frac{1}{n}) - 2\sum_{x\epsilon X_1} \mu_B(x) + 2\sum_{x\epsilon X_2} \mu_B(x)\right) \\ &= \frac{1}{n+1}\left(-2\sum_{x\epsilon X_1} \mu_B(x) + 2\sum_{x\epsilon X_2} \mu_B(x)\right) \\ &= \frac{1}{n+1}\left(2\sum_{x\epsilon X_2} \mu_B(x) - 2\sum_{x\epsilon X_1} \mu_B(x)\right). \end{aligned}$$

□

Example 4.4. *Let* $X = \{x_i \mid i = 1, 2, ..., 6\}$. *Let* $A(x_i) = i$, $i = 1, 2, ..., 6$. *Then* $\mu_A(x_i) = \frac{i}{6}, i = 1, 2, ..., 6$. *Let* $B(x_1) = 6, B(x_2) = 5, B(x_3) = 3, B(x_4) = 4, B(x_5) = 2, B(x_6) = 1$. *It follows that*

$$\mu_{A^*}(x_i) = 1 - \mu_A(x_i) + \frac{1}{6}, i = 1, 2, ..., 6.$$

Thus

$$\begin{aligned} \sum_{x\epsilon X} |\mu_A(x) - \mu_B(x)| &= \frac{1}{6}(5 + 3 + 0 + 0 + 3 + 5) = \frac{16}{6}, \\ \sum_{x\epsilon X} |\mu_{A^*}(x) - \mu_B(x)| &= \frac{1}{6}(0 + 0 + 1 + 1 + 0 + 0) = \frac{2}{6}. \end{aligned}$$

$$\begin{aligned} S(\mu_A, \mu_B) &= 1 - \frac{16}{6}\frac{1}{7} = \frac{42 - 16}{42} = \frac{26}{42}, \\ S(\mu_{A^*}, \mu_B) &= 1 - \frac{2}{6}\frac{1}{7}. = \frac{42 - 2}{42} = \frac{40}{42} \end{aligned}$$

Hence

$$S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) = \frac{26}{42} - \frac{40}{42} = -\frac{14}{42} = -\frac{1}{3}.$$

Using Corollary 4.3,we get

$$\begin{aligned} & S(\mu_A, \mu_B) - S(\mu_{A^*}, \mu_B) \\ = & \frac{1}{n+1}\left(2\sum_{x\epsilon X_2}\mu_B(x) - 2\sum_{x\epsilon X_1}\mu_B(x)\right) \\ = & \frac{1}{7}\cdot\frac{1}{6}(2(4+2+1) - 2(6+5+3)) \\ = & \frac{1}{7}\cdot\frac{1}{6}\cdot 2(7-14) = -\frac{1}{3}. \end{aligned}$$

5. Conclusion

We determined the fuzzy similarity measures of rankings of U. S. states with respect to domestic violence, female homicide, and sexual violence against teens. We found that these measures were low. We then determined the fuzzy similarity measures of the best place for women to live and the above issues concerning violence We found the fuzzy similarity measures to be medium. We then developed theoretical results to compare the fuzzy similarity measure of two rankings with the fuzzy similarity measure of one of the rankings and the reverse ranking of the other.

6. Author Contributions

Coceptualization, methodology J N Mordeson and D S Malik resources , J N Mordeson and D S Malik; writing—original draft preparation, validation, J N Mordeson; writing—review and editing,Sunil Mathew; Conceptualization, Sunil Mathew.

7. Data Availability Statement

Data used in the work is from references 1,2,5 and 6.

8. Funding

No funding was received for the work.

9. Conflict of interest

The authors declare no conflict of interest.

10. **Appendix A**

Table 7. Violence

State	Domestic Violence	Female Homicide	Sexual Violence Against Teens
Alabama	24		20
Alaska	3	1	29
Arizona	4	11	31
Arkansas	10	9	32
California	35	28	34
Colorado	28	12	
Connecticut	21	41	
Delaware	22	38	1
Florida	20		14
Georgia	25	31	13
Hawaii	36	48	6
Idaho	46	32	35
Illinois	8	44	19
Indiana	5	30	
Iowa	33	47	7
Kansas	39	16	15
Kentucky	1	25	2
Louisiana	31	5	
Maine	17	34	
Maryland	38	24	5
Massachusetts	40	46	4
Michigan	30	15	30
Minnesota	41	45	
Mississippi	14	18	11
Missouri	7	7	

Table 8. Violence(Continued)

State	Domestic Violence	Female Homicide	Sexual Violence Against Teens
Montana	26	29	24
Nebraska	43	40	25
Nevada	2	3	8
New Hampshire	37	4	21
New Jersey	32	42	28
New Mexico	23	2	18
New York	48	35	12
North Carolina	34	27	22
North Dakota	49	37	9
Ohio	19	36	
Oklahoma	11	8	23
Oregon	13	23	
Pennsylvania	27	39	27
Rhode Island	47	26	
South Carolina	6	6	17
South Dakota	50	19	16
Tennessee	15	10	
Texas	12	20	10
Utah	44	33	33
Vermont	18	13	
Virginia	45	14	3
Washington	9	22	
West Virginia	16	17	26
Wisconsin	29	21	
Wyoming	42	43	

TABLE 9. Best state to live

State	Rank	State	Rank
Alabama	47	Montana	31
Alaska	27	Nebraska	18
Arizona	30	Nevada	34
Arkansas	48	New Hampshire	5
California	14	New Jersey	10
Colorado	13	New Mexico	39
Connecticut	2	New York	7
Delaware	21	North Carolina	32
Florida	29	North Dakota	19
Georgia	36	Ohio	24
Hawaii	9	Oklahoma	41
Idaho	38	Oregon	17
Illinois	12	Pennsylvania	16
Indiana	33	Rhode Island	4
Iowa	22	South Carolina	43
Kansas	25	South Dakota	28
Kentucky	46	Tennessee	44
Louisiana	50	Texas	40
Maine	8	Utah	35
Maryland	6	Vermont	3
Massachusetts	1	Virginia	26
Michigan	20	Washington	23
Minnesota	11	West Virginia	45
Mississippi	49	Wisconsin	15
Missouri	37	Wyoming	42

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