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SOLVABLE INTRANSITIVE PERMUTATION GROUPS WITH CONSTANT MOVEMENT

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Abstract. In this paper, all solvable intransitive permutation groups with constant movement are classified and we show that they are one of the following groups: a cyclic p-group, an elementary abelian p-group, a Frobenius group of order 12 or a Frobenius group of order pq, where p and q are primes such that $p = q(q-1)+1$.

Keywords: Permutation group, Orbit, Constant movement, Frobenius group.
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1. Introduction

The exploration of permutation groups is a long-standing field with a fascinating background that dates back to the early 19th century, during the emergence of group theory. The concept of a group was first introduced by Galois in his examination of the permutations of polynomial equation roots (leading to the well-known Galois group of the polynomial). Throughout much of the 19th century, there was significant focus on groups of substitutions, now commonly referred to as permutation groups. Permutation group theory has interesting applications in other areas of mathematics, such as combinatorics, representation theory, number theory, graph theory, etc. Let G be a permutation group on a set Ω without any fixed points in Ω. If the size $|\Gamma^g - \Gamma|$ is bounded for a subset Γ of Ω and for $g \in G$ then the movement of Γ is defined as

$$\text{move}(\Gamma) := \sup_g |\Gamma^g - \Gamma|.$$

If there exists $m \in \mathbb{N}$ such that $\text{move}(\Gamma) \leq m$ for all $\Gamma \subseteq \Omega$ then we say that G has bounded movement m. Moreover, the movement of G is defined as

$$\text{move}(G) := \sup_{\Gamma, g} |\Gamma^g - \Gamma|.$$

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The notion of movement was first introduced in [8]. In the same way, we can define the movement of g for each $g \in G$, as

$$\mathrm{move}(g) := \sup_{\Gamma} |\Gamma^g - \Gamma|.$$

By Theorem 1 of [8], if G has bounded movement equal to m then Ω is finite. Also, both the number of G-orbits in Ω and the length of each of them are bounded above by linear functions of m. So, an upper bound for $|\Omega|$ is obtained. In [3], that upper bound was improved for $p \geq 5$, where p is the least odd prime divisor of $|G|$. If the movements of all non-identity elements of G are equal then we say that G has constant movement. Obviously, if G has constant movement then it has bounded movement. The concept of constant movement was firstly introduced in [2] and moreover, solvable intransitive permutation groups G with constant movement m having maximum degree were classified there. Some years later in [4], 2-transitive permutation groups with abelian stabilizers having constant movement were investigated. Also, primitive permutation groups with constant movement were determined in [5]. In this paper, we continue these works and classify all solvable intransitive permutation groups G with constant movement m and prove the Theorem 1.1. We denote by $K \rtimes H$ a semidirect product of K and H with normal subgroup K and for a real number x, $\lfloor x \rfloor$ is the integer part of x and $\lceil x \rceil$ is the least integer greater than or equal to x.

Theorem 1.1. *Let G be a solvable intransitive permutation group on a set Ω that has no fixed points on Ω. Let G have constant movement m. Then one of the following holds:*
(i) G is either a cyclic p-group or $G \cong \mathbb{Z}_p^d$, for some p prime and $d \in \mathbb{N}$;
(ii) $G \cong \mathbb{Z}_2^2 \rtimes \mathbb{Z}_3$;
(iii) $G \cong \mathbb{Z}_p \rtimes \mathbb{Z}_q$, where p and q are primes such that $p = q(q-1)+1$.

The structure of the groups in Theorem 1.1 will be explained in Examples 2.2, 2.4 and 2.5.

2. Preliminaries and Examples

Let G be the permutation group acting on a finite set Ω and $1 \neq g \in G$. Suppose that $g = \prod_{i=1}^{s} c_i$ is the disjoint cycle representation of g with $|c_i| = l_i \geq 2$ for all $1 \leq i \leq s$. Assume that each cycle c_i has the representation $c_i = (c_{i_1} c_{i_2} \cdots c_{i_{l_i}})$. Suppose that for each i, we choose $\lfloor l_i/2 \rfloor$ points from the cycle c_i and put them in a set $\Gamma(g)$, such that $\Gamma(g)^g \cap \Gamma(g) = \emptyset$. For instance, choose

$$\Gamma(g) := \{c_{1_2}, c_{1_4}, \ldots, c_{1_{h_1}}, c_{2_2}, c_{2_4}, \ldots, c_{2_{h_2}}, ..., c_{s_2}, c_{s_4}, \ldots, c_{s_{h_s}}\},$$

where $h_i = l_i - 1$ if l_i is odd and $h_i = l_i$ if l_i is even. In this case, $\Gamma(g)$ is the set of every second point of each cycle of g. Note that the determination of $\Gamma(g)$

is not unique, because it depends on the way each cycle is represented. By the description of $\Gamma(g)$, we have

$$|\Gamma(g)^g - \Gamma(g)| = |\Gamma(g)| = \sum_{i=1}^{s} \lfloor l_i/2 \rfloor.$$

It was shown in Lemma 2.1 of [7] that the above value is an upper bound for $|\Gamma^g - \Gamma|$, when Γ is an arbitrary subset of Ω.

As a straightforward consequence, we have the following result.

Lemma 2.1. *Let G be a permutation group on a set Ω. If $g \in G$ has a cycle decomposition $g = \prod_{i=1}^{s} c_i$, where the cycle c_i has length l_i, then* $\text{move}(g) = \sum_{i=1}^{s} \lfloor l_i/2 \rfloor$.

Proof. For each $\Gamma \subseteq \Omega$, the inequality $|\Gamma^g - \Gamma| \leq \sum_{i=1}^{s} \lfloor l_i/2 \rfloor$ holds by Lemma 2.1 of [7]. Hence, $\text{move}(g) \leq \sum_{i=1}^{s} \lfloor l_i/2 \rfloor$. As $\text{move}(g) \geq |\Gamma(g)^g - \Gamma(g)|$, we conclude the desired result. □

Now in the following examples, we explain the structure of the groups appearing in Theorem 1.1.

Example 2.2. *Let p be a prime, d a natural number, $G := \mathbb{Z}_p^d$, $t := (p^d - 1)/(p-1)$, and $H_1, \dots, H_t$ be the subgroups of index p in G. Consider $\Omega_i := \{H_i g \mid g \in G\}$ be the right cosets of H_i in G and $\Omega := \Omega_1 \cup \cdots \cup \Omega_t$. Since each non-identity element $g \in G$ lies in $(p^{d-1} - 1)/(p-1)$ of the subgroups H_i, the action of it as a permutation on Ω has $p(p^{d-1} - 1)/(p-1)$ fixed points and p^{d-1} orbits of length p. So, $move(g) = p^{d-1}(p-1)/2 := m$ if p is odd and 2^{d-1} if $p = 2$. Therefore, G has constant movement equal to m.*
Note that by Theorem 1.2 of [2], this family of groups attains the maximum degree $n = \lceil 2mp/(p-1) \rceil + t - 1$.

Before giving the second example, it is necessary to state a remark.

Remark 2.3. ([1]) Let G act as a permutation group on a set Ω and $g \in G$. Assume that $\Gamma \subseteq \Omega$ and Ω is the disjoint union of G-invariant sets Ω_1 and Ω_2. Set $\Gamma_i := \Gamma \cap \Omega_i$ and suppose that for $i = 1, 2$, g_i be the permutation on Ω_i induced by g, respectively. Since $|\Gamma^g - \Gamma| = |\Gamma_1^{g_1} - \Gamma_1| + |\Gamma_2^{g_2} - \Gamma_2|$, we have

$$\text{move}_\Omega(g) = \sum_{i=1}^{2} \max\{|\Gamma_i^{g_i} \setminus \Gamma_i||\Gamma_i \subseteq \Omega_i\} = \text{move}_{\Omega_1}(g_1) + \text{move}_{\Omega_2}(g_2).$$

Example 2.4. *Let $G_1 := \mathbb{Z}_2^2$ and $G_2 := \mathbb{Z}_3$ be permutation groups on the sets $\Omega_1 = \{1, 2, 3, 4\}$ and $\Omega_2 = \{5, 6, 7\}$ respectively, where $G_1 = \langle (1\,2)(3\,4), (1\,3)(2\,4) \rangle$ and $G_2 = \langle (1\,2\,3)(5\,6\,7) \rangle$. Set $\Omega := \Omega_1 \cup \Omega_2$. Then $G := G_1 \rtimes G_2$ acts on Ω as* *a permutation group with* $t = 2$ *orbits. Also, every non-identity element of* G *splits into two cycles of length* 2 *or into two cycles of length* 3*. Therefore,* G *has constant movement* $m = 2$*. Note that,* G *acts Frobeniusly on the set* Ω_1*.*

Example 2.5. *Let* q *and* $p := q(q-1)+1$ *be two odd prime numbers and* G *be a Frobenius group of order* pq *with kernel* $K \cong \mathbb{Z}_p$ *acting on a set* Ω_1 *of size* p*. Suppose that* Ω_2 *is a set of size* q *and* β *is a cycle of length* q *in it. Assume that* $\gamma := \beta\gamma_1 \cdots \gamma_{q-1}$*, where* γ_i $(1 \leq i \leq q-1)$ *are* $q-1$ *disjoint cycles of length* q *on* Ω_1*. Let* $H = \langle\gamma\rangle$ *and* $K = \langle\alpha\rangle$*, where* α *is a cycle of length* p *in* Ω_1*. Then* $H \cong \mathbb{Z}_q$ *and, for an integer* r*, the group* $G := K \rtimes H$ *given by the following relations:*

$$\alpha^p = \gamma^q = 1, \quad \gamma^{-1}\alpha\gamma = \alpha^r, \quad r^q \equiv 1 (\mathrm{mod}\ p);$$

is a permutation group on $\Omega := \Omega_1 \cup \Omega_2$ *with* $t = 2$ *orbits such that* $\mathbb{Z}_p$ *acts regularly on* Ω_1*. Since each non-identity element of* G *has* q *cycles of length* q *or one cycle of length* p*,* G *has constant movement* $m = \frac{q(q-1)}{2}$*. Obviously,* G *in its action on the set* Ω_1 *is a Frobenius group.*
In particular, for $q = 3$*, we can consider* $\Omega_1 = \{1, ..., 7\}$*,* $\Omega_2 = \{1', 2', 3'\}$*,* $\alpha = (1 \cdots 7)$ *and* $\gamma = (1'2'3')(2\,3\,5)(4\,7\,6)$*. Then,* $G := \langle\alpha, \gamma\rangle \cong \mathbb{Z}_7 \rtimes \mathbb{Z}_3$ *is a permutation group with* $t = 2$ *orbits on a set* $\Omega := \Omega_1 \cup \Omega_2$ *of size* 10*, in which every* $1 \neq g \in G$ *has* 3 *cycles of size* 3 *or is a cycle of size* 7 *and therefore, in both cases, it has movement* 3*.*

3. Main results

Let G be a solvable intransitive permutation group on a set Ω with $\mathrm{fix}(G) = \emptyset$ and having constant movement. The cycle structure of elements of a permutation group with constant movement is known by [2].

Lemma 3.1. *(Lemma 3.6 of [2]) Let* G *be a permutation group on a set* Ω *of size* n *with constant movement* m *and let* $1 \neq g \in G$*. Then, all non-trivial cycles of* g *have the same size and the order of* g *is either an odd prime or a power of* 2*.*

Now, we give two definitions which are needed in the rest of the paper.

Definition 3.2. A group G, in which every non-identity element has prime power order, is called an EPPO-group.
For example, consider the alternating group A_6. The orders of elements of this group are 2,3,4,5. So, it is an EPPO-group.

The complete classification of finite EPPO-groups is given in Lemma 0.4 of [9].

Definition 3.3. Suppose that G is a finite group and there exists a normal series $1 \lhd N \lhd K \lhd G$ of G such that N is the Frobenius kernel of K and $\frac{K}{N}$ is the Frobenius kernel of $\frac{G}{N}$. Then, G is called a 2-Frobenius group.
One of the known 2-Frobenius groups is the symmetric group S_4. Consider

the normal series $1 \lhd \mathbb{Z}_2^2 \lhd A_4 \lhd S_4$. Then, one can easily check that S_4 is 2-Frobenius.

Before starting the proof of Theorem 1.1, we state the basic facts for Frobenius groups.

Lemma 3.4. *[6] Let G be a Frobenius group with kernel K and complement H. Then,*
(i) K is nilpotent. Also, if $|H|$ is even then K is abelian.
(ii) Sylow p-subgroups of H are cyclic for odd primes p and are cyclic or generalized quaternion groups for $p = 2$.

Note that the converse of the above lemma is not necessarily true. For example, consider the Dicyclic group Dic_5 of order 20, which is the semidirect product of the cyclic groups $\mathbb{Z}_5$ and $\mathbb{Z}_4$. This group satisfies conditions (i) and (ii), but is not a Frobenius group, because the center of this group is of size 2.

Now we are ready to start the proof of Theorem 1.1.
Let G be a solvable intransitive permutation group with constant movement m and $t > 1$ orbits on a set Ω of size n which has no fixed points on Ω. By Lemma 3.1, G is an EPPO-group. If G is solvable then, by Lemma 0.4 of [9], G is a p-group, a Frobenius group or a 2-Frobenius group and the order of G is of the form $p^\alpha q^\beta$ for two primes p and q and integers α and β. We discuss each case separately. First, suppose that G is a p-group.

Proposition 3.5. *G is a p-group with constant movement if and only if it is either cyclic or the group described in Example 2.2.*

Proof. Suppose that d is a natural number and G is a p-group of order p^d with constant movement m acting on a set Ω. Let $|\Omega| = n$ and let $t > 1$ be the number of G-orbits.
First, we consider the case where $p = 2$. If G is cyclic then it is isomorphic to $\mathbb{Z}_{2^d} = \langle x_1 \cdots x_t \rangle$, where each x_i is a cycle of length 2^d. So, it has t orbits and has constant movement $m = t2^{d-1}$. Assume that G is non-cyclic. Let $g \in G$. If k is the length of a nontrivial cycle in the cycle decomposition of g and s is the number of $\langle g \rangle$-orbits on Ω then

$$l := |\mathrm{supp}(g)| = sk = 2s(\frac{k}{2}) = 2s\lfloor \frac{k}{2} \rfloor = 2\,\mathrm{move}(g) = 2m.$$

So, every non-identity element in G moves exactly the same number of points in Ω. Thus, $m = l/2$ and $|\mathrm{fix}(g)| = n - l$. By Burnside's lemma,

$$t = \frac{1}{|G|} \sum_{g \in G} |\mathrm{fix}(g)| = \frac{1}{2^d}(n + (2^d - 1)(n - l)) = n - l + \frac{l}{2^d}. \quad (1)$$

So, $|G| = 2^d \mid l$. If G is an elementary abelian 2-group then G is isomorphic to the direct product of cyclic groups of order 2, say G_i. Suppose that each G_i acts on a set Ω_i of size $2t_i$. Thus, we have $t = t_1 + \cdots + t_d$ and $n = 2t$. By (1), we conclude that $2^d t = l(2^d - 1)$. Since $t \leq 2^d - 1$, we have $l \leq 2^d$. Therefore,

$l = 2^d$, $t = 2^d - 1$, $m = 2^{d-1}$, $n = 2(2^d - 1)$ and G is the group mentioned in Example 2.2.
Suppose now that G is not an elementary abelian 2-group. Also, suppose that s is a natural number and G is isomorphic to the product of t copies of a non-abelian 2-group H of order 2^s in its regular representation. So, G acts on a set Ω of size $n = 2^s t$ and has movement $m = 2^{s-1}t$. By (1), $2^s(2^{st}-1) = (2^s-1)2^{st}$. But, this equality holds if and only if $t = 1$, which is not possible.
Now assume that G is a p-group ($p \neq 2$) with constant movement m. By Lemma 3.1, G is of exponent p. If G is cyclic then $G \cong \mathbb{Z}_p = \langle x_1 \cdots x_t \rangle$, where each x_i is a cycle of length p. So, it has t orbits and has constant movement $m = t(p-1)/2$. Assume that G is abelian and non-cyclic. Then, G is an elementary abelian p-group of order p^d. By a similar argument mentioned above, we conclude that $l = p^d$, $t = (p^d - 1)/(p-1)$, $m = p^{d-1}(p-1)/2$, $n = p(p^d - 1)/(p-1)$ and G is the group mentioned in Example 2.2. Finally, suppose that G is non-abelian. Suppose that s is a natural number and G is isomorphic to the product of t copies of a non-abelian p-group H of order p^s in its regular representation. So, G acts on a set Ω of size $n = p^s t$. By a similar argument, we have $l = p^s t$ and $p^s(p^{st} - 1) = (p^s - 1)p^{st}$. But, this equality holds only for $t = 1$, which is impossible.

Conversely, if G is the group described in Example 2.2 then it was shown there that G has constant movement $m = p^{d-1}(p-1)/2$. Also, it is shown in the above argument that if G is a cyclic p-group then it has constant movement. □

Proposition 3.6. *Let G be a Frobenius group with constant movement. Then,*
(i) If G has even order then it is the group mentioned in Example 2.4.
(ii) If G has odd order then it is the group mentioned in Example 2.5.

Proof. (i) Let G be a Frobenius group of even order and $K := O_2(G)$ and $P := O_p(G)$ be the largest normal 2-subgroup and normal p-subgroup of G, respectively. Then, by Lemma 0.4 of [9], G is isomorphic to groups $K \rtimes \mathbb{Z}_{p^\alpha}$, $P \rtimes \mathbb{Z}_{2^\beta}$ or $P \rtimes Q_{2^\beta}$ for some natural numbers α, β. Note that Q_{2^β} has only regular faithful representation.
Case 1: Suppose that $G \cong K \rtimes \mathbb{Z}_{p^\alpha}$. Since G has constant movement, it can not have elements of orders $p^2, p^3, \ldots, p^\alpha$. So, in this case, $G \cong K \rtimes \mathbb{Z}_p$. We prove that $p = 3$ and $K \cong \mathbb{Z}_2^2$. Assume that K is a permutation group on a set Ω_1 of size 2^β and $\Omega_2, \ldots, \Omega_t$ be the other orbits of G such that $|\Omega_2| = \cdots = |\Omega_t| = p$. Set $\Omega := \bigcup_{i=1}^t \Omega_i$. Then,

$$t2^\beta p = \sum_{g \in G} |\text{fix}(g)| = 2^\beta + (t-1)p + (2^\beta - 1)(t-1)p + (2^\beta p - 2^\beta)(\frac{2^\beta - 1}{p}).$$

This equality holds if and only if $2^\beta = p + 1$. Let g and h be a 2-element and a p-element of G, respectively. Since G has constant movement m,

$$2^{\beta-1} = \text{move}(g) = \text{move}(h) = (t-1)\frac{p-1}{2} + \frac{2^\beta - 1}{p}\frac{p-1}{2} = t\frac{(p-1)}{2}.$$

Thus, $p+1=2^\beta=t(p-1)$. Since $t=\frac{p+1}{p-1}$ is an integer, $p=3$, $t=2$ and so, $G\cong \mathbb{Z}_2^2\rtimes\mathbb{Z}_3$ as defined in Example 2.4 that is the case (ii) of Theorem 1.1.
Case 2: Suppose that $G\cong P\rtimes\mathbb{Z}_{2^\beta}$ or $G\cong P\rtimes Q_{2^\beta}$ for some natural number β. Since P is of exponent p, we conclude that $P\cong\mathbb{Z}_p^d$ for some natural number d by Lemma 3.4. Since G has constant movement, $\frac{p^{d-1}(p-1)}{2}=2^{\beta-1}$. But this happens only when $d=1$ and $p=2^\beta+1$. So, $G\cong\mathbb{Z}_p\rtimes\mathbb{Z}_{p-1}$ or $G\cong\mathbb{Z}_p\rtimes Q_{p-1}$. By a similar argument mentioned above, we conclude that $2^{\beta-1}=t\frac{(p-1)}{2}$. It implies that $t=1$, which is a contradiction. Therefore, there is no group which satisfies in this case.
(ii) Suppose that $G=P\rtimes Q$ is a Frobenius group of odd order $p^\alpha q^\beta$. By part(2) of Lemma 0.4 of [9], Q is cyclic. Since G doesn't have elements of orders $q^2,q^3,...,q^\beta$, we have $|Q|=q$. Suppose that $Q=\langle y\rangle$, where $y=y_1\cdots y_k$ and $|y_i|=q$ for all $1\le i\le k$. Since G has constant movement and P acts regularly on a set Ω_1 of size p^α, $p^{\alpha-1}(\frac{p-1}{2})=k\frac{q-1}{2}$. On the other side, we have $p^\alpha-1=(k-1)q$. So, $k-q=p^{\alpha-1}-1$. Assume that $q-1=u$. Then, $k=u+p^{\alpha-1}$ and this implies that $(u+p^{\alpha-1})u=p^\alpha-p^{\alpha-1}$. But the equation $u^2+p^{\alpha-1}u-(p^\alpha-p^{\alpha-1})=0$ has solution if $\Delta=p^{2(\alpha-1)}+4p^\alpha-4p^{\alpha-1}$ is a square integer. If $\alpha>3$ then $(p^{\alpha-1}+2p-3)^2<\Delta<(p^{\alpha-1}+2p-2)^2$, which is not possible. If $\alpha=3$ then $\Delta=p^4+4p^3-4p^2=p^2(p^2+4p-4)$ can not be a square. If $\alpha=2$ then $\Delta=6p^2-4p\equiv 2p^2\equiv 2 \pmod 4$, which is not possible. So, $\alpha=1$ and we have $(k-1)q=k(q-1)$. But this equality holds if $k=q$ and therefore, $p=q(q-1)+1$, $m=\frac{q(q-1)}{2}$ and G has two orbits, one of them is of length $p=q(q-1)+1$ and the other is of length q. So, G is a Frobenius group with kernel $\mathbb{Z}_p$ and complement $\mathbb{Z}_q$, in which $\mathbb{Z}_q$ is generated by q cycles of length q are described in Example 2.5. So, the case (iii) of Theorem 1.1 is attained. □

Proposition 3.7. *There is no 2-Frobenius group with constant movement.*

Proof. Suppose that $G=PQR$ is a 2-Frobenius group with constant movement m. Since $P\rtimes Q$ and $Q\rtimes R$ are Frobenius groups with constant movement, they are isomorphic to groups $\mathbb{Z}_2^2\rtimes\mathbb{Z}_3$ or $\mathbb{Z}_p\rtimes\mathbb{Z}_p$ (for $p=q(q-1)+1$) by preceding argument. By part(2) of Lemma 0.4 of [9], we see that the first case does not occur. Suppose that $P\rtimes Q\cong\mathbb{Z}_p\rtimes\mathbb{Z}_q$ and $Q\rtimes R\cong\mathbb{Z}_q\rtimes\mathbb{Z}_r$ in which p,q,r are primes such that $p=q(q-1)+1$ and $q=r(r-1)+1$. Since $r=p$ or $r=q$, we have $p(p^2-p+1)=1$ or $q^2-2q+1=0$ that is impossible. Therefore, there is no group satisfying in this case. □

Now, Theorem 1.1 follows from Propositions 3.5-3.7.

4. Conclusion

In this paper, we investigate solvable intransitive permutation groups with constant movement. We show that they are one of the following groups: a cyclic p-group, an elementary abelian p-group $\mathbb{Z}_p^d$, the Frobenius group $\mathbb{Z}_2^2\rtimes\mathbb{Z}_3$

or the Frobenius group $\mathbb{Z}_p \rtimes \mathbb{Z}_q$, where p and q are two primes such that $p = q(q-1)+1$. The structure of these groups are described in Examples 2.2, 2.4 and 2.5.
For future works, one can use parts (3) and (4) of Lemma 0.4 of [9] to verify non-solvable intransitive permutation groups with constant movement. It was stated there that if G is simple, then $G \cong L_2(q)$, $q = 5, 7, 8, 9, 17$; $L_3(4)$, $Sz(8)$ or $Sz(32)$ and if G is non-solvable and non-simple, $G \cong M_{10}$, or G has an elementary abelian 2-subgroup P, P is normal in G and $G/P \cong L_2(q)$, $q = 5, 8$; $Sz(8)$ or $Sz(32)$.

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