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CONDITION PSEUDOSPECTRUM IN NON-ARCHIMEDEAN BANACH SPACES

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Abstract. In this article, we introduce and study the condition pseudospectrum of bounded linear operator pencils on non-Archimedean Banach spaces. We obtain a characterization of the condition pseudospectrum of bounded linear operator pencils on non-Archimedean Banach spaces, the relation between the condition pseudospectrum of a bounded linear operator pencil and the pseudospectrum of this operator pencil in a non-Archimedean valued field is investigated. Finally, we characterize the essential spectrum of bounded linear operator pencils by non-Archimedean completely continuous linear operators, and we illustrate our work with some examples.

Keywords: Non-Archimedean Banach spaces, pseudospectrum, condition pseudospectrum, operator pencils.
2020 MSC: 47A10, 47S10, 46S10.

1. **Introduction**

In complex operator theory, L. N. Trefethen and M. Embree [14] developed the study of pseudospectra of bounded linear operators on complex Banach spaces. Recently, A. Ammar, A. Jeribi and K. Mahfoudhi [2] demonstrated several properties on the condition pseudospectrum of bounded linear operator pencils $A-\lambda B$ when B is not necessarily invertible. Also, they gave some results concerning the condition pseudospectra of bounded linear operator pencils on complex Banach spaces.

In non-Archimedean operator theory, A. Ammar et al. [1] initiated the study of the pseudospectra and the essential pseudospectra of linear operators in non-Archimedean Banach spaces and in non-Archimedean Hilbert spaces, respectively. In particular, they characterized these pseudospectra. Furthermore, inspired by T. Diagana and F. Ramaroson [4], they established a relationship between the essential pseudospectrum of a closed linear operator and the essential pseudospectrum of this closed linear operator perturbed by completely continuous linear operators in non-Archimedean Hilbert spaces. Recently, A. Ammar et al. [3] extended and studied the condition pseudospectra and the essential

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condition pseudospectra of bounded linear operators on non-Archimedean Banach spaces. On the other hand, J. Ettayb [8] introduced and studied the structured pseudospectrum and the structured essential pseudospectrum of closed linear operator pencils on non-Archimedean Banach spaces. The pseudospectra of matrix pencils with entries in a non-Archimedean field were introduced by J. Ettayb [10]. Moreover, J. Ettayb [9] introduced and studied the structured pseudospectrum and the structured Fredholm pseudospectrum of bounded linear operator pencils on non-Archimedean Banach spaces. In particular, he proved several results on the structured pseudospectrum and the structured Fredholm pseudospectrum of bounded linear operator pencils on non-Archimedean Banach spaces. Recently, J. Ettayb [7] established that the essential pseudospectrum of operator pencils is invariant under perturbation of completely continuous linear operators on a non-Archimedean Banach space over a spherically complete field $\mathbb{K}$, and he proved a characterization of the essential pseudospectrum of operator pencils. Furthermore, he introduced and studied the (n, ε)-pseudospectrum of bounded linear operators and operator pencils on non-Archimedean Banach spaces.

In [11], J. Ettayb introduced and studied the bounded linear operator pencils, the pseudospectrum and the essential pseudospectrum of bounded linear operator pencils on non-Archimedean Banach spaces. He proved that the essential pseudospectrum of a bounded linear operator pencil is invariant under perturbation of compact operators on non-Archimedean Banach spaces over the field of p-adic numbers $\mathbb{Q}_p$. Finally, he gave a characterization of the essential pseudospectrum of bounded linear operator pencils by means of the spectra of all perturbed compact operators in non-Archimedean Banach spaces.

The pseudospectra of operators played an important role in non-Archimedean functional analysis based on the theory of non-Archimedean Banach spaces which has received a lot of attention due to substantial differences between the latter and classical cases, see [1, 3–5].

The generalized eigenvalue problem is one of the interesting problems in non-Archimedean operator theory which played a central role in many parts of non-Archimedean applied mathematics and physics. For more details, see [9–11]. This work is motivated by many studies on non-Archimedean spectral theory and perturbation theory of bounded linear operators. For further details, see [1, 3, 7–11]. Indeed, suppose that X is a non-Archimedean Banach space over a non-Archimedean field $\mathbb{K}$, $A, B : X \longrightarrow X$ are two bounded linear operators, $\varepsilon > 0, \lambda \in \mathbb{K}$ and $y \in X$. Consider the operator equation

$$(A - \lambda B)x = y. \tag{1}$$

Setting $\sigma(A, B) = \{\lambda \in \mathbb{K} : A - \lambda B \text{ is not invertible in } \mathcal{L}(X)\}$ and $\Lambda_\varepsilon(A, B) = \sigma(A, B) \cup \{\lambda \in \mathbb{K} : \|A - \lambda B\| \|(A - \lambda B)^{-1}\| > \varepsilon^{-1}\}$, we have the following:

(i) If $\lambda \notin \sigma(A, B)$, then the equation (1) is solvable;
(ii) If $\lambda \notin \Lambda_\varepsilon(A, B)$, then the equation (1) has a stable solution.

In view of this, the condition spectrum is expected to be a useful tool in the numerical solution of operator equations.

Throughout this paper, $\mathbb{K}$ is a complete non-Archimedean non-trivially valued field with valuation $|\cdot|$, $\mathbb{Q}_p$ is the field of p-adic numbers, $\mathbb{R}_+$ is the set of nonnegative real numbers, X is a non-Archimedean Banach space over $\mathbb{K}$ and $\mathcal{L}(X)$ denotes the collection of all bounded linear operators on X. Recall that $\mathbb{K}$ is said to be spherically complete if the intersection of every decreasing sequence of balls is nonempty [4,13]. If $A \in \mathcal{L}(X)$, then $N(A)$ and $R(A)$ denote the kernel and the range of A, respectively. The set $\sigma(A,B) = \{\lambda \in \mathbb{K} : A - \lambda B \text{ is not invertible in } \mathcal{L}(X)\}$ is called the spectrum of the pencil (A,B) of the form $A - \lambda B$. The resolvent set $\rho(A,B)$ is given $\rho(A,B) = \mathbb{K} \backslash \sigma(A,B)$.

In this paper, we introduce and study the condition pseudospectrum and the essential condition pseudospectrum of operator pencils on non-Archimedean Banach spaces. We characterize the condition pseudospectrum of operator pencils on non-Archimedean Banach spaces. The relation between the condition pseudospectrum and the pseudospectrum of operator pencils in non-Archimedean valued fields is given. Finally, we characterize the essential spectrum of operator pencils by completely continuous linear operators on non-Archimedean Banach spaces.

2. Preliminaries

We continue by recalling some preliminaries.

Definition 2.1. [4] A field $\mathbb{K}$ is said to be non-Archimedean if it is endowed with an absolute value $|\cdot| : \mathbb{K} \to \mathbb{R}_+$ which satisfies the following conditions:

(i) $|\lambda| = 0$ if and only if $\lambda = 0$;
(ii) For all $\lambda, \mu \in \mathbb{K}$, $|\lambda\mu| = |\lambda||\mu|$;
(iii) For each $\lambda, \mu \in \mathbb{K}$, $|\lambda + \mu| \leq \max\{|\lambda|, |\mu|\}$.

Definition 2.2. [4] Let X be a vector space over a complete non-Archimedean field $\mathbb{K}$. A mapping $\|\cdot\| : X \to \mathbb{R}_+$ is called a non-Archimedean norm if the following hold:

(i) For all $u \in X$, $\|u\| = 0$ if and only if $u = 0$;
(ii) For any $u \in X$ and $\mu \in \mathbb{K}$, $\|\mu u\| = |\mu|\|u\|$;
(iii) For all $u, v \in X$, $\|u + v\| \leq \max(\|u\|, \|v\|)$.

Definition 2.3. [4] A non-Archimedean normed space in which every Cauchy sequence is a convergent sequence is called a non-Archimedean Banach space.

Definition 2.4. [7] Let X be a non-Archimedean Banach space over a complete non-Archimedean field $\mathbb{K}$. For a pair (A,B) of operators in $\mathcal{L}(X)$, the spectrum $\sigma(A,B)$ of a bounded linear operator pencil (A,B) of the form $A-\lambda B$ is defined by

$$\begin{aligned}\sigma(A,B) &= \{\lambda \in \mathbb{K} : A - \lambda B \quad \text{is not invertible in } \mathcal{L}(X)\} \\ &= \{\lambda \in \mathbb{K} :\ 0 \in \sigma(A - \lambda B)\}.\end{aligned}$$

The resolvent set $\rho(A,B)$ of a bounded linear operator pencil (A,B) of the form $A-\lambda B$ is the complement of $\sigma(A,B)$ in $\mathbb{K}$ given by

$$\rho(A,B)=\{\lambda\in\mathbb{K}:R_\lambda(A,B)=(A-\lambda B)^{-1}\text{ exists in }\mathcal{L}(X)\}.$$

$R_\lambda(A,B)$ is called the resolvent of a bounded linear operator pencil (A,B) of the form $A-\lambda B$.

Definition 2.5. [7] Let $A,B\in\mathcal{L}(X)$ and $\varepsilon>0$. The pseudospectrum $\sigma_\varepsilon(A,B)$ of a bounded linear operator pencil (A,B) of the form $A-\lambda B$ on X is given by

$$\sigma_\varepsilon(A,B)=\sigma(A,B)\cup\{\lambda\in\mathbb{K}:\|(A-\lambda B)^{-1}\|>\varepsilon^{-1}\}.$$

The pseudoresolvent $\rho_\varepsilon(A,B)$ of a bounded linear operator pencil (A,B) of the form $A-\lambda B$ is defined by

$$\rho_\varepsilon(A,B)=\rho(A,B)\cap\{\lambda\in\mathbb{K}:\|(A-\lambda B)^{-1}\|\leq\varepsilon^{-1}\},$$

by convention $\|(A-\lambda B)^{-1}\|=\infty$ if and only if $\lambda\in\sigma(A,B)$.

Definition 2.6. [13] Let $A\in\mathcal{L}(X)$. Then A is said to have finite rank if $\dim R(A)$ is finite.

Definition 2.7. [7] Let $A,B\in\mathcal{L}(X)$ such that $A\neq B$ and B be a non-null operator. The essential spectrum $\sigma_e(A,B)$ of (A,B) is given by

$$\sigma_e(A,B)=\{\lambda\in\mathbb{K}:A-\lambda B\text{ is not a Fredholm operator of index }0\}.$$

Definition 2.8. [4] Let X be a non-Archimedean Banach space and let $A\in\mathcal{L}(X)$. Then A is said to be completely continuous if there is a sequence of finite rank linear operators $(A_n)_{n\in\mathbb{N}}$ such that $\|A_n-A\|\to 0$ as $n\to\infty$.

$\mathcal{C}_c(X)$ is the collection of all completely continuous operators on X.

Definition 2.9. [12] Let $A\in\mathcal{L}(X)$. Then A is called a Fredholm operator if $R(A)$ is closed, $\alpha(A)=\dim N(A)$ and $\beta(A)=\dim(X/R(A))$ are finite. The collection of all Fredholm operators on X is denoted by $\Phi(X)$. Let $A\in\Phi(X)$, the index $ind(A)$ of A is defined by $ind(A)=\alpha(A)-\beta(A)$.

For more details on non-Archimedean Fredholm operators and completely continuous operators, we refer to [4], [12] and [13]. Let X be a non-Archimedean Banach space over a spherically complete field $\mathbb{K}$, the following lemmas hold.

Lemma 2.10. *[6] If $A\in\mathcal{L}(X)$, then $R(A)^\perp=N(A^*)$.*

Lemma 2.11. *[5] If $A\in\Phi(X)$ and $K\in\mathcal{C}_c(X)$, then $A+K\in\Phi(X)$.*

Lemma 2.12. *[3] If $A\in\Phi(X)$, then for all $K\in\mathcal{C}_c(X)$, we have $ind(A+K)=ind(A)$.*

From Lemma 4.11 and Lemma 4.13 of [12], we have the following lemma.

Lemma 2.13. *If $x_1^*, \cdots, x_n^*$ are linearly independent vectors in X^*, then there are vectors $x_1, \cdots, x_n$ in X such that*

$$x_j^*(x_k) = \delta_{j,k} = \begin{cases} 1 \text{ if } j = k \\ 0 \text{ if } j \neq k \end{cases} \quad 1 \leq j, k \leq n. \tag{2}$$

Moreover, if $x_1, \cdots, x_n$ are linearly independent vectors in X, then there are vectors $x_1^*, \cdots, x_n^*$ in X^* such that (2) holds.

Theorem 2.14. *[13] Let X be a non-Archimedean Banach space over a spherically complete field $\mathbb{K}$. Then for all $z \in X \backslash \{0\}$, there exists a linear functional $\xi \in X^*$ such that $\xi(z) = 1$ and $\|\xi\| = \|z\|^{-1}$.*

3. Condition pseudospectrum of operator pencils on non-Archimedean spaces

In this section, we define the condition pseudospectra of bounded operator pencils on non-Archimedean Banach spaces. We have the following definition.

Definition 3.1. Let X be a non-Archimedean Banach space over $\mathbb{K}$, let $A, B \in \mathcal{L}(X)$ and $\varepsilon > 0$. The condition pseudospectrum of a bounded linear operator (A, B) of the form $A - \lambda B$ on X is defined by

$$\Lambda_\varepsilon(A, B) = \sigma(A, B) \cup \{\lambda \in \mathbb{K} : \|A - \lambda B\| \|(A - \lambda B)^{-1}\| > \varepsilon^{-1}\}$$

by convention $\|A - \lambda B\| \|(A - \lambda B)^{-1}\| = \infty$ if and only if $\lambda \in \sigma(A, B)$.
The pseudoresolvent of a bounded linear operator (A, B) of the form $A - \lambda B$ is given by

$$\rho(A, B) \cap \{\lambda \in \mathbb{K} : \|A - \lambda B\| \|(A - \lambda B)^{-1}\| \leq \varepsilon^{-1}\}.$$

Proposition 3.2. *Let X be a non-Archimedean Banach space over $\mathbb{K}$, let $A, B \in \mathcal{L}(X)$ and $\varepsilon > 0$. Then*

(i) $\sigma(A, B) = \bigcap\limits_{\varepsilon > 0} \Lambda_\varepsilon(A, B)$;

(ii) *If $0 < \varepsilon_1 \leq \varepsilon_2$, then $\sigma(A, B) \subset \Lambda_{\varepsilon_1}(A, B) \subset \Lambda_{\varepsilon_2}(A, B)$.*

Proof. (i) By Definition 3.1, we have for all $\varepsilon > 0, \sigma(A, B) \subset \Lambda_\varepsilon(A, B)$. Then $\sigma(A, B) \subseteq \bigcap\limits_{\varepsilon > 0} \Lambda_\varepsilon(A, B)$. Conversely, let $\lambda \in \bigcap\limits_{\varepsilon > 0} \Lambda_\varepsilon(A, B)$. Then for all $\varepsilon > 0, \lambda \in \Lambda_\varepsilon(A, B)$. If $\lambda \notin \sigma(A, B)$, then $\lambda \in \{\lambda \in \mathbb{K} : \|A - \lambda B\| \|(A - \lambda B)^{-1}\| > \varepsilon^{-1}\}$. Taking the limit as $\varepsilon \to 0^+$, we get $\|A - \lambda B\| \|(A - \lambda B)^{-1}\| = \infty$. Thus $\lambda \in \sigma(A, B)$.

(ii) Let $0 < \varepsilon_1 \leq \varepsilon_2$ and let $\lambda \in \Lambda_{\varepsilon_1}(A, B)$. Consequently, $\|A - \lambda B\| \|(A - \lambda B)^{-1}\| > \varepsilon_1^{-1} \geq \varepsilon_2^{-1}$. Hence $\lambda \in \Lambda_{\varepsilon_2}(A, B)$.

□

Lemma 3.3. *Let X be a non-Archimedean Banach space over $\mathbb{K}$, let $A, B \in \mathcal{L}(X)$ and $\varepsilon > 0$. Then $\lambda \in \Lambda_\varepsilon(A,B) \setminus \sigma(A,B)$ if and only if there is an element $x \in X \setminus \{0\}$ such that*

$$\|(A - \lambda B)x\| < \varepsilon \|A - \lambda B\| \|x\|.$$

Proof. If $\lambda \in \Lambda_\varepsilon(A,B) \setminus \sigma(A,B)$, then

$$\|A - \lambda B\| \|(A - \lambda B)^{-1}\| > \varepsilon^{-1}.$$

Hence

$$\|(A - \lambda B)^{-1}\| > \frac{1}{\varepsilon \|A - \lambda B\|}.$$

Moreover

$$\sup_{y \in X \setminus \{0\}} \frac{\|(A - \lambda B)^{-1} y\|}{\|y\|} > \frac{1}{\varepsilon \|A - \lambda B\|}.$$

Then there exists an element $y \in X \setminus \{0\}$ such that $\|(A - \lambda B)^{-1} y\| > \frac{\|y\|}{\varepsilon \|A - \lambda B\|}$. Setting $x = (A - \lambda B)^{-1} y$, we obtain

$$\|(A - \lambda B)^{-1}(A - \lambda B)x\| > \frac{\|(A - \lambda B)x\|}{\varepsilon \|A - \lambda B\|}.$$

Hence

$$\|(A - \lambda B)x\| < \varepsilon \|A - \lambda B\| \|x\|.$$

Conversely, suppose that there is $x \in X \setminus \{0\}$ such that

$$\|(A - \lambda B)x\| < \varepsilon \|A - \lambda B\| \|x\|.$$

If $\lambda \notin \sigma(A,B)$ and $x = (A - \lambda B)^{-1} y$, then $\|x\| \leq \|(A - \lambda B)^{-1}\| \|y\|$. Hence $\|x\| < \varepsilon \|(A - \lambda B)^{-1}\| \|A - \lambda B\| \|x\|$. Thus

$$\|(A - \lambda B)^{-1}\| \|A - \lambda B\| > \frac{1}{\varepsilon}.$$

Consequently, $\lambda \in \Lambda_\varepsilon(A,B) \setminus \sigma(A,B)$. □

Theorem 3.4. *Let X be a non-Archimedean Banach space over $\mathbb{K}$. Let $A, B \in \mathcal{L}(X)$ be invertible operators and $\varepsilon > 0$. Set $k = \|A^{-1}\| \|A\| \|B^{-1}\| \| \|B\|$. If $\lambda \in \Lambda_\varepsilon(A^{-1}, B) \setminus \{0\}$, then $\frac{1}{\lambda} \in \Lambda_{\varepsilon k}(A, B^{-1}) \setminus \{0\}$.*

Proof. For $\lambda \in \Lambda_\varepsilon(A^{-1},\ B)\backslash\{0\}$, we have

$$\begin{aligned}
\frac{1}{\varepsilon} &< \|A^{-1} - \lambda B\|\|(A^{-1} - \lambda B)^{-1}\| \\
&= \|B\lambda\Big(\frac{B^{-1}}{\lambda} - A\Big)A^{-1}\|\|\lambda^{-1}A\Big(\frac{B^{-1}}{\lambda} - A\Big)^{-1}B^{-1}\| \\
&\leq |\lambda|\|A^{-1}\|\|B^{-1}\|\|B\|\|\Big(\frac{B^{-1}}{\lambda} - A\Big)\| \\
&\quad \times\|\Big(\frac{B^{-1}}{\lambda} - A\Big)^{-1}\||\lambda|^{-1}\|A\| \\
&= \|A^{-1}\|\|A\|\|B^{-1}\|\|B\|\|\Big(\frac{B^{-1}}{\lambda} - A\Big)\| \\
&\quad \times\|\Big(\frac{B^{-1}}{\lambda} - A\Big)^{-1}\|.
\end{aligned}$$

Hence, $\frac{1}{\lambda} \in \Lambda_{\varepsilon k}(A,\ B^{-1})\backslash\{0\}$. □

Theorem 3.5. *Let X be a non-Archimedean Banach space over $\mathbb{K}$, $A, B \in \mathcal{L}(X)$, $\lambda \in \mathbb{K}$ and $\varepsilon > 0$. If there exists $C \in \mathcal{L}(X)$ such that $\|C\| < \varepsilon\|A - \lambda B\|$ and $\lambda \in \sigma(A + C, B)$, then $\lambda \in \Lambda_\varepsilon(A, B)$.*

Proof. Suppose that there is an operator $C \in \mathcal{L}(X)$ such that $\|C\| < \varepsilon\|A-\lambda B\|$ and $\lambda \in \sigma(A + C, B)$. If $\lambda \notin \Lambda_\varepsilon(A, B)$, then $\lambda \in \rho(A, B)$ and $\|A - \lambda B\|\|(A - \lambda B)^{-1}\| \leq \varepsilon^{-1}$.
Consider D defined on X by

$$D = \sum_{n=0}^{\infty}(A - \lambda B)^{-1}\Big(- C(A - \lambda B)^{-1}\Big)^n.$$

Consequently, $D = (A - \lambda B)^{-1}(I + C(A - \lambda B)^{-1})^{-1}$. Hence, for all $y \in X$, $D(I + C(A - \lambda B)^{-1})y = (A - \lambda B)^{-1}y$. Putting $x = (A - \lambda B)^{-1}y$, we obtain

$$\text{for all } x \in X, D(A - \lambda B + C)x = x$$

and

$$\text{for each } x \in X,\ (A - \lambda B + C)Dx = x.$$

Then $A-\lambda B+C$ is invertible which is a contradiction. Thus $\lambda \in \Lambda_\varepsilon(A, B)$. □

Now, we characterize the condition pseudospectra $\Lambda_\varepsilon(A, B)$ of the pencil (A, B) as follows.

Theorem 3.6. *Let X be a non-Archimedean Banach space over a spherically complete field $\mathbb{K}$ with $\|X\| \subseteq |\mathbb{K}|$, $A, B \in \mathcal{L}(X)$ and $\varepsilon > 0$. Then*

$$\Lambda_\varepsilon(A, B) = \bigcup_{C\in\mathcal{C}_\varepsilon(X)} \sigma(A + C, B)$$

where $\mathcal{C}_\varepsilon(X) = \{C \in \mathcal{L}(X) : \|C\| < \varepsilon\|A - \lambda B\|\}$.

Proof. Let $\lambda \in \bigcup_{C\in\mathcal{C}_\varepsilon(X)} \sigma(A+C,B)$. We argue by contradiction. Suppose that $\lambda \in \rho(A,B)$ and $\|A-\lambda B\|\|(A-\lambda B)^{-1}\| \leq \varepsilon^{-1}$. Consider D defined on X by

$$D=\sum_{n=0}^{\infty}(A-\lambda B)^{-1}\Big(-C(A-\lambda B)^{-1}\Big)^{n}.$$

Consequently, $D=(A-\lambda B)^{-1}(I+C(A-\lambda B)^{-1})^{-1}$. Hence, for all $y\in X$, $D(I+C(A-\lambda B)^{-1})y=(A-\lambda B)^{-1}y$. Putting $x=(A-\lambda B)^{-1}y$, we deduce that $D(A-\lambda B+C)x=x$ for all $x\in X$. Moreover

$$\text{for each } x\in X,\ (A-\lambda B+C)Dx=x.$$

Hence, we conclude that $A-\lambda B+C$ is invertible and $D=(A-\lambda B+C)^{-1}$. Conversely, assume that $\lambda\in\Lambda_\varepsilon(A,B)$. If $\lambda\in\sigma(A,B)$, we may put $C=0$. If $\lambda\in\Lambda_\varepsilon(A,B)$ and $\lambda\notin\sigma(A,B)$, then by applying Lemma 3.3 and $\|X\|\subseteq|\mathbb{K}|$, there exists $x\in X\backslash\{0\}$ such that $\|x\|=1$ and $\|(A-\lambda B)x\|<\varepsilon\|A-\lambda B\|$. By Theorem 2.14, there exists $\varphi\in X^*$ such that $\varphi(x)=1$ and $\|\varphi\|=\|x\|^{-1}=1$. Consider C defined on X by for all $y\in X$, $Cy=-\varphi(y)(A-\lambda B)x$. Hence, $\|C\|<\varepsilon\|A-\lambda B\|$ and $D(C)=X$. Thus C is bounded. Moreover for $x\in X\backslash\{0\}, (A-\lambda B+C)x=0$. So, $A-\lambda B+C$ is not invertible. Consequently, $\lambda\in\bigcup_{C\in\mathcal{C}_\varepsilon(X)}\sigma(A+C,B)$. □

Example 3.7. *Let $\mathbb{K}=\mathbb{Q}_p$, $a,b,c,d\in\mathbb{Q}_p$ with $a\neq b$ and $c\neq d$ and $c,d\neq 0$. If*

$$A=\begin{pmatrix} a & 0\\ 0 & b\end{pmatrix}$$

and

$$B=\begin{pmatrix} c & 0\\ 0 & d\end{pmatrix},$$

we have $\sigma(A,B)=\{\frac{a}{c},\ \frac{b}{d}\}$. Then

$$\|A-\lambda B\|=\max\{|a-\lambda c|,|b-\lambda d|\}.$$

Hence

$$\|(A-\lambda B)^{-1}\|=\max\Big\{\frac{1}{|a-\lambda c|},\frac{1}{|b-\lambda d|}\Big\}.$$

Thus

$$\Lambda_\varepsilon(A,B)=\{\frac{a}{c},\ \frac{b}{d}\}\cup\{\lambda\in\mathbb{Q}_p:\ \frac{|a-\lambda c|}{|b-\lambda d|}>\frac{1}{\varepsilon}\Big\}\cup\{\lambda\in\mathbb{Q}_p:\ \frac{|b-\lambda d|}{|a-\lambda c|}>\frac{1}{\varepsilon}\Big\}.$$

Example 3.8. *If*

$$A=\begin{pmatrix} 1 & 1\\ 1 & 1\end{pmatrix}\in\mathcal{M}_2(\mathbb{K})$$

and

$$B=\begin{pmatrix} 1 & 1\\ 0 & 1\end{pmatrix}\in\mathcal{M}_2(\mathbb{K}),$$

we have $\sigma(A,B)=\{0,1\}$. *One can see that*

$$\|A-\lambda B\|=\max\{|1-\lambda|,1\}$$

and

$$\|(A-\lambda B)^{-1}\|=\max\Big\{\frac{1}{|\lambda|},\frac{1}{|\lambda(1-\lambda)|}\Big\}.$$

Then

$$\Lambda_\varepsilon(A,B)=\{0,1\}\cup\{\lambda\in\mathbb{K}:\max\{|1-\lambda|,1\}\max\Big\{\frac{1}{|\lambda|},\frac{1}{|\lambda(1-\lambda)|}\Big\}>\frac{1}{\varepsilon}\}.$$

Set $l^\infty=\{x=(x_i)_{i\in\mathbb{N}}\in\mathbb{K}^{\mathbb{N}}:\sup_{i\in\mathbb{N}}|x_i|<\infty\}$ and it is equipped with the norm $x=(x_i)_{i\in\mathbb{N}}\in l^\infty,\|x\|=\sup_{i\in\mathbb{N}}|x_i|$, see [4]. We have the following examples.

Example 3.9. *Suppose that* $\mathbb{K}=\mathbb{Q}_p$. *Consider* $A,B:l^\infty\longrightarrow l^\infty$ *defined by*

$$A(x_1,x_2,\cdots)=(x_1,p^2x_2,p^3x_3,\cdots,p^nx_n,0,\cdots)$$

and

$$B(x_1,x_2,\cdots)=(x_1,x_2,\cdots,x_n,0,\cdots).$$

Then $\Lambda_\varepsilon(A,B)=\sigma(A,B)=\mathbb{Q}_p$.

Example 3.10. *Suppose that* $\mathbb{K}=\mathbb{Q}_p$ *and* $p\neq 2$. *Consider* $A,B:l^\infty\longrightarrow l^\infty$ *defined by* $A(x_1,x_2,\cdots)=(x_2,0,0,\cdots)$ *and* $B(x_1,x_2,\cdots)=(2x_1,x_2,x_3,\cdots)$. *Then* $\sigma(A,B)=\{0\}$ *and for* $\lambda\neq 0$,

$$(\lambda B-A)^{-1}=(\frac{x_1}{2\lambda}+\frac{x_2}{2\lambda^2},\frac{x_2}{\lambda},\frac{x_3}{\lambda},\cdots),\quad \|(\lambda B-A)^{-1}\|=\max\{\frac{1}{|\lambda|},\frac{1}{|\lambda|^2}\}$$

and

$$\|\lambda B-A\|=\max\{1,|\lambda|\}.$$

Then

$$\Lambda_\varepsilon(A,B)=\{0\}\cup\{\lambda\in\mathbb{Q}_p:|\lambda|<\sqrt{\varepsilon}\}.$$

Example 3.11. *Suppose that* $\mathbb{K}=\mathbb{Q}_p$ *where* $p\neq 3$. *If*

$$A=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}\in\mathcal{M}_2(\mathbb{Q}_p)$$

and

$$B=\begin{pmatrix}3&0&0\\0&1&0\\0&0&1\end{pmatrix}\in\mathcal{M}_2(\mathbb{Q}_p),$$

we have $\sigma(A,B)=\{0\}$. *One can see that for* $\lambda\neq 0$,

$$(\lambda B-A)^{-1}=\begin{pmatrix}\frac{1}{3\lambda}&\frac{1}{3\lambda^2}&0\\0&\frac{1}{\lambda}&0\\0&0&\frac{1}{\lambda}\end{pmatrix}.$$

Then
$$\|A-\lambda B\| = \max\{1, |\lambda|\}$$
and
$$\|(A-\lambda B)^{-1}\| = \max\Big\{\frac{1}{|\lambda|}, \frac{1}{|\lambda|^2}\Big\}.$$
Consequently
$$\Lambda_\varepsilon(A,B) = \{0\} \cup \{\lambda \in \mathbb{Q}_p : |\lambda| < \sqrt{\varepsilon}\}.$$

Proposition 3.12. *Let X be a non-Archimedean Banach space over $\mathbb{K}$, $A, B \in \mathcal{L}(X), \lambda \in \mathbb{K}$ such that $\|A-\lambda B\| \neq 0$ and $\varepsilon > 0$. Then*

(i) *$\lambda \in \Lambda_\varepsilon(A,B)$ if and only if $\lambda \in \sigma_{\varepsilon\|A-\lambda B\|}(A,B)$.*
(ii) *$\lambda \in \sigma_\varepsilon(A,B)$ if and only if $\lambda \in \Lambda_{\frac{\varepsilon}{\|A-\lambda B\|}}(A,B)$.*

Proof.
(i) Let $\lambda \in \Lambda_\varepsilon(A,B)$. Then $\lambda \in \sigma(A,B)$ and $\|A-\lambda B\|\|(A-\lambda B)^{-1}\| > \varepsilon^{-1}$. Hence $\lambda \in \sigma(A,B)$ and $\|(A-\lambda B)^{-1}\| > \frac{1}{\varepsilon\|A-\lambda B\|}$. Consequently $\lambda \in \sigma_{\varepsilon\|A-\lambda B\|}(A,B)$. The converse is similar.
(ii) Let $\lambda \in \sigma_\varepsilon(A,B)$. Then $\lambda \in \sigma(A,B)$ and $\|(A-\lambda B)^{-1}\| > \varepsilon^{-1}$. Thus
$$\lambda \in \sigma(A,B) \text{ and } \|A-\lambda B\|\|(A-\lambda B)^{-1}\| > \frac{\|A-\lambda B\|}{\varepsilon}.$$
Then $\lambda \in \sigma_{\frac{\varepsilon}{\|A-\lambda B\|}}(A,B)$. The converse is similar. □

Proposition 3.13. *Let X be a non-Archimedean Banach space over $\mathbb{K}$, $A, B \in \mathcal{L}(X)$ and $\varepsilon > 0$. If $\alpha, \beta \in \mathbb{K}$ with $\beta \neq 0$, then $\Lambda_\varepsilon(\beta A+\alpha B, B) = \alpha + \beta\Lambda_\varepsilon(A,B)$.*

Proof. Let $\lambda \in \Lambda_\varepsilon(\beta A+\alpha B, B)$ and $\beta \neq 0$. Then $\lambda \in \sigma(\beta A+\alpha B, B)$ and
$$\begin{aligned}\frac{1}{\varepsilon} &< \|\beta A+\alpha B-\lambda B\|\|(\beta A+\alpha B-\lambda B)^{-1}\| \\ &= \|\Big(A-\frac{\lambda-\alpha}{\beta}B\Big)\|\|\Big(A-\frac{\lambda-\alpha}{\beta}B\Big)^{-1}\|.\end{aligned}$$
Hence $\frac{\lambda-\alpha}{\beta} \in \Lambda_\varepsilon(A,B)$. Then $\lambda \in \alpha + \beta\Lambda_\varepsilon(A,B)$. However, the opposite inclusion follows by symmetry. □

Proposition 3.14. *Let X be a non-Archimedean Banach space over $\mathbb{K}$, $A, B \in \mathcal{L}(X)$ such that $AB = BA$ and $B^{-1} \in \mathcal{L}(X)$ and $\varepsilon > 0$. Then*
$$\Lambda_{\frac{\varepsilon}{\|B\|\|B^{-1}\|}}(B^{-1}A, I) \subset \Lambda_\varepsilon(A,B).$$

Proof. If $\lambda \in \Lambda_{\frac{\varepsilon}{\|B\|\|B^{-1}\|}}(B^{-1}A, I)$, then
$$\begin{aligned}\frac{\|B\|\|B^{-1}\|}{\varepsilon} &< \|B^{-1}A-\lambda I\|\|(B^{-1}A-\lambda I)^{-1}\| \\ &= \|B^{-1}\Big(A-\lambda B\Big)\|\|\Big(A-\lambda B\Big)^{-1}B\| \\ &\leq \Big(\|B\|\|B^{-1}\|\Big)\|\Big(A-\lambda B\Big)\|\|\Big(A-\lambda B\Big)^{-1}\|.\end{aligned}$$

Thus $\lambda \in \Lambda_\varepsilon(A,B)$. Consequently $\Lambda_{\frac{\varepsilon}{\|B\|\|B^{-1}\|}}(B^{-1}A, I) \subset \Lambda_\varepsilon(A,B)$. $\square$

Theorem 3.15. *Let X be a non-Archimedean Banach space over $\mathbb{K}$, $A, B, C, U \in \mathcal{L}(X)$ such that $UB = BU$ and there is U^{-1}. If $C = U^{-1}AU$, then for all $\varepsilon > 0$, $k = \|U^{-1}\|\|U\|$, we have*

$$\Lambda_{\frac{\varepsilon}{k^2}}(C,B) \subseteq \Lambda_\varepsilon(A,B) \subseteq \Lambda_{k^2\varepsilon}(C,B).$$

Proof. Let $\lambda \in \Lambda_{\frac{\varepsilon}{k^2}}(C,B)$, then $\lambda \in \sigma(C,B)\Big(=\sigma(A,B)\Big)$ and

$$\begin{aligned}\frac{k^2}{\varepsilon} < \|C-\lambda B\|\|(C-\lambda B)^{-1}\| &= \|U^{-1}\Big(A-\lambda B\Big)U\|\|U^{-1}\Big(A-\lambda B\Big)^{-1}U\| \\ &\leq \Big(\|U\|\|U^{-1}\|\Big)^2\|\Big(A-\lambda B\Big)\|\|\Big(A-\lambda B\Big)^{-1}\| \\ &\leq k^2\|\Big(A-\lambda B\Big)\|\|\Big(A-\lambda B\Big)^{-1}\|.\end{aligned}$$

Since $k^2 > 0$, we have $\lambda \in \Lambda_\varepsilon(A,B)$. Hence $\Lambda_{\frac{\varepsilon}{k^2}}(C,B) \subseteq \Lambda_\varepsilon(A,B)$.
Let $\lambda \in \Lambda_\varepsilon(A,B)$. Then $\lambda \in \sigma(C,B)\Big(=\sigma(A,B)\Big)$ and

$$\begin{aligned}\frac{1}{\varepsilon} < \|A-\lambda B\|\|(A-\lambda B)^{-1}\| &= \|U\Big(C-\lambda B\Big)U^{-1}\|\|U\Big(C-\lambda B\Big)^{-1}U^{-1}\| \\ &\leq \Big(\|U\|\|U^{-1}\|\Big)^2\|\Big(C-\lambda B\Big)\|\|\Big(C-\lambda B\Big)^{-1}\| \\ &\leq k^2\|\Big(C-\lambda B\Big)\|\|\Big(C-\lambda B\Big)^{-1}\|.\end{aligned}$$

Hence $\lambda \in \Lambda_{k^2\varepsilon}(C,B)$. Consequently $\Lambda_\varepsilon(A,B) \subseteq \Lambda_{k^2\varepsilon}(C,B)$. $\square$

Set $\mathbb{E}_\omega = \{x = (x_i)_{i\in\mathbb{N}} : \forall i \in \mathbb{N}, x_i \in \mathbb{K}$ and $\lim_{i\to\infty}|\omega_i|^{\frac{1}{2}}|x_i| = 0\}$ and it is equipped with the norm $x = (x_i)_{i\in\mathbb{N}} \in \mathbb{E}_\omega$, $\|x\| = \sup_{i\in\mathbb{N}}(|\omega_i|^{\frac{1}{2}}|x_i|)$, see [4]. We have the following example.

Example 3.16. *Let $A, B \in \mathcal{L}(\mathbb{E}_\omega)$ be two diagonal operators such that for each $i \in \mathbb{N}$, $Ae_i = a_ie_i$, $Be_i = b_ie_i$ where $(a_i)_{i\in\mathbb{N}}, (b_i)_{i\in\mathbb{N}} \subset \mathbb{K}$ with $\sup\limits_{i\in\mathbb{N}}|a_i|, \sup\limits_{i\in\mathbb{N}}|b_i| < \infty$ and for all $i \in \mathbb{K}$, $b_i^{-1} \neq 0$, $\sup\limits_{i\in\mathbb{N}}|b_i|^{-1}$ is finite and $\lim\limits_{i\to\infty}|b_i|^{-1} \neq 0$. One can see that*

$$\sigma(A,B) = \{\frac{a_i}{b_i} : i \in \mathbb{N}\}.$$

For all $\lambda \in \rho(A,B)$, we have

$$\begin{aligned}\|(A-\lambda B)^{-1}\| &= \sup_{i\in\mathbb{N}}\frac{\|(A-\lambda B)^{-1}e_i\|}{\|e_i\|} \\ &= \sup_{i\in\mathbb{N}}\left|\frac{1}{a_i-\lambda b_i}\right| = \frac{1}{\inf\limits_{i\in\mathbb{N}}|a_i-\lambda b_i|}\end{aligned}$$

and $\|A-\lambda B\| = \sup\limits_{i\in\mathbb{N}}|a_i-\lambda b_i|$. *Hence*

$$\left\{\lambda\in\mathbb{K} : \|A-\lambda B\|\|(A-\lambda B)^{-1}\| > \frac{1}{\varepsilon}\right\} = \left\{\lambda\in\rho(A,B) : \frac{\sup\limits_{i\in\mathbb{N}}|a_i-\lambda b_i|}{\inf\limits_{i\in\mathbb{N}}|a_i-\lambda b_i|} > \frac{1}{\varepsilon}\right\}.$$

Consequently

$$\sigma_\varepsilon(A,B) = \{\frac{a_i}{b_i} : i\in\mathbb{N}\}\cup\{\lambda\in\rho(A,B) : \inf_{i\in\mathbb{N}}|a_i-\lambda b_i| < \varepsilon\}$$

and

$$\Lambda_\varepsilon(A,B) = \{\frac{a_i}{b_i} :\ i\in\mathbb{N}\}\cup\left\{\lambda\in\rho(A,B) : \frac{\sup\limits_{i\in\mathbb{N}}|a_i-\lambda b_i|}{\inf\limits_{i\in\mathbb{N}}|a_i-\lambda b_i|} > \frac{1}{\varepsilon}\right\}.$$

Now, we characterize the essential spectrum $\sigma_e(A,B)$ of (A,B) in $\mathbb{K}$.

Theorem 3.17. *Let X be a non-Archimedean Banach space over a spherically complete field $\mathbb{K}$. Let $A,B\in\mathcal{L}(X)$ such that A^*,B^* exist. Then*

$$\sigma_e(A,B) = \bigcap_{K\in\mathcal{C}_c(X)}\sigma(A+K,B).$$

Proof. If $\lambda\notin\bigcap\limits_{K\in\mathcal{C}_c(X)}\sigma(A+K,B)$, then there is $K\in\mathcal{C}_c(X)$ such that $\lambda\in\rho(A+K,B)$. Hence

$$A+K-\lambda B\in\Phi(X)$$

and

$$ind(A+K-\lambda B) = 0.$$

Moreover

$$A-\lambda B = A+K-\lambda B-K.$$

Since $K\in\mathcal{C}_c(X)$, applying Lemmas 2.11 and 2.12, we obtain

$$A-\lambda B\in\Phi(X)$$

and

$$ind(A-\lambda B) = 0.$$

Thus $\lambda\notin\sigma_e(A,B)$. Consequently

$$\sigma_e(A,B)\subseteq\bigcap_{K\in\mathcal{C}_c(X)}\sigma(A+K,B).$$

Conversely, let $\lambda\in\mathbb{K}$ such that $A-\lambda B\in\Phi(X)$ and $ind(A-\lambda B)=0$. Put $\alpha(A-\lambda B)=\beta(A-\lambda B)=n$. Let $\{x_1,\cdots,x_n\}$ be a basis for $N(A-\lambda B)$ and $\{y_1^*,\cdots,y_n^*\}$ be a basis for $R(A-\lambda B)^\perp$. By Lemma 2.13, there are functionals $x_1^*,\cdots,x_n^*$ in X^* $\big(X^*$ is the dual space of $X\big)$ and elements $y_1,\cdots,y_n$ in X such that

$$x_j^*(x_k)=\delta_{j,k} \textit{ and } y_j^*(y_k)=\delta_{j,k},\ 1\le j,\ k\le n,$$

where $\delta_{j,k} = 0$ if $j \neq k$ and $\delta_{j,k} = 1$ if $j = k$. Consider K defined on X by

$$\begin{array}{rrl} K: & X \to & X \\ & x \mapsto & \displaystyle\sum_{i=1}^{n} x_i^*(x) y_i. \end{array}$$

One can see that K is linear and $D(K) = X$. In fact for all $x \in X$,

$$\begin{aligned} \|Kx\| &= \|\sum_{i=1}^{n} x_i^*(x) y_i\| \\ &\leq \max_{1 \leq i \leq n} \|x_i^*(x) y_i\| \\ &\leq \max_{1 \leq i \leq n} (\|x_i^*\| \|y_i\|) \|x\|. \end{aligned}$$

Hence K is a finite rank operator. Thus K is completely continuous. We demonstrate that

$$N(A - \lambda B) \cap N(K) = \{0\} \tag{3}$$

and

$$R(A - \lambda B) \cap R(K) = \{0\}. \tag{4}$$

Let $x \in N(A - \lambda B) \cap N(K)$. Hence

$$x = \sum_{i=1}^{n} \alpha_i x_i \ \text{ with } \alpha_1, \cdots, \alpha_n \in \mathbb{K}.$$

Then for all $1 \leq j \leq n$, $x_j^*(x) = \displaystyle\sum_{i=1}^{n} \alpha_i \delta_{i,j} = \alpha_j$. If $x \in N(K)$, then $Kx = 0$. Hence

$$\sum_{j=1}^{n} x_j^*(x) y_j = 0.$$

Therefore for all $1 \leq j \leq n$, $x_j^*(x) = 0$. Hence $x = 0$. Consequently

$$N(A - \lambda B) \cap N(K) = \{0\}.$$

Let $y \in R(A-\lambda B) \cap R(K)$. Then $y \in R(A-\lambda B)$ and $y \in R(K)$. Let $y \in R(K)$. We have

$$y = \sum_{i=1}^{n} \alpha_i y_i \ \text{ with } \alpha_1 \cdots \alpha_n \in \mathbb{K}.$$

Then for each $1 \leq j \leq n$, $y_j^*(y) = \displaystyle\sum_{i=1}^{n} \alpha_i \delta_{i,j} = \alpha_j$. Now, if $y \in R(A-\lambda B)$, then for all $1 \leq j \leq n$, $y_j^*(y) = 0$. Thus $y = 0$. Therefore

$$R(A - \lambda B) \cap R(K) = 0.$$

On the other hand, $K \in \mathcal{C}_c(X)$. Hence we deduce from Lemma 2.11 that $A - \lambda B + K \in \Phi(X)$ and $ind(A + K - \lambda B) = 0$. Thus

$$\alpha(A + K - \lambda B) = \beta(A + K - \lambda B). \tag{5}$$

If $x \in N(A + K - \lambda B)$, then $(A - \lambda B)x = -Kx$ in $R(A - \lambda B) \cap R(K)$. It follows from (4) that $(A - \lambda B)x = -Kx = 0$. Hence $x \in N(A - \lambda B) \cap N(K)$ and from (3), we have $x = 0$. Thus $\alpha(A+K-\lambda B) = 0$. It follows from (5) that $R(A + K - \lambda B) = X$. Consequently $A - \lambda B + K$ is invertible and we conclude that $\lambda \notin \bigcap_{K \in \mathcal{C}_c(X)} \sigma(A + K, B)$. □

We finish with the following example.

Example 3.18. *Suppose that $\mathbb{K} = \mathbb{Q}_p$. Consider $A, B : l^\infty \longrightarrow l^\infty$ defined by $A(x_1, x_2, \cdots) = (x_1, x_2, \cdots)$ and $B(x_1, x_2, \cdots) = (x_2, x_3, \cdots, x_n, 0, \cdots)$. Then*

$$\sigma_e(A, B) = \sigma(A, B) = \emptyset.$$

Consequently, for any $K \in \mathcal{C}_c(l^\infty)$, we have

$$\bigcap_{K \in \mathcal{C}_c(X)} \sigma(A + K, B) = \emptyset.$$

4. Conclusion

In this article, we introduced and studied the condition pseudospectrum of continuous linear operator pencils on non-Archimedean Banach spaces. We established several results on the condition pseudospectra of operator pencils. In addition, we provided several illustrative examples.

In the future, we will study the condition pseudospectrum and the essential condition pseudospectrum of closed, densely defined linear operator pencils on non-Archimedean Banach spaces.

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