Shahid Bahonar University of Kerman
Journal of Mahani Mathematical Research
Print ISSN: 2251-7952 Online ISSN: 2645-4505

FUZZY FUNCTION APPROXIMATION FOR MULTI-CHOICE GOAL PROGRAMMING IN TRANSPORTATION PROBLEMS

Z. Arami, M. Arabameri ✉, and H. Mishmast Nehi

Article type: Research Article
(Received: 21 June 2025, Received in revised form 10 January 2026)
(Accepted: 17 May 2026, Published Online: 18 May 2026)

Abstract. Transportation problems are widely used decision making models in logistics, production, and supply chain management. In real world applications, the input parameters such as costs, supplies, and demands are often uncertain or imprecise, making classical crisp formulations inadequate. To address this challenge, this study proposes a fuzzy multi choice goal programming (FMCGP) model enhanced with fuzzy function approximation techniques. Unlike previous works, where fuzzy transportation problems are treated using direct defuzzification or ranking approaches, our method integrates fuzzy least-squares linear regression and a fuzzy binary polynomial approximation to represent and approximate multi choice fuzzy goals flexibly. This dual approach allows the decision maker to simultaneously handle multiple fuzzy objectives and constraints within a unified framework. A key feature of the proposed methodology is that all comparisons between fuzzy and crisp values are evaluated using the necessity measure with a degree of 0.8, ensuring mathematically consistent and practically interpretable inequality relations. To demonstrate the model's applicability, we present a case study of a transportation planning problem under uncertainty. The numerical experiments illustrate how the proposed approach outperforms existing fuzzy transportation methods in terms of solution feasibility, interpretability, and computational efficiency. The results confirm that the FMCGP model with fuzzy function approximation provides a powerful and flexible tool for decision-making under uncertainty, offering improved accuracy and robustness compared with classical fuzzy transportation approaches. In addition, the framework is general enough to be extended to other types of fuzzy optimization problems beyond transportation.

Keywords: Fuzzy Transportation Problem, Multi-Choice Goal Programming, Fuzzy least squares linear approach.
2020 MSC: 90C29, 90C70, 90B06, 41A10.

1. Introduction

The transportation problem (TP) is widely recognized as one of the key decision-making and optimization matters with numerous applications in real-world scenarios. As a special case of linear programming problems, TP aims to

✉ arabameri@math.usb.ac.ir, ORCID: 0000-0003-0269-4547
https://doi.org/10.22103/jmmr.2026.25444.1820

Publisher: Shahid Bahonar University of Kerman
How to cite: Z. Arami, M. Arabameri, H. Mishmast Nehi, *Fuzzy function approximation for multi-choice goal programming in transportation problems*, J. Mahani Math. Res. 2026; 15(2): 457-476.

determine the optimal distribution of goods from various sources to specified destinations, such that the total transportation cost is minimized and the demand requirements are fulfilled. The classical transportation problem was first presented and formulated by Hitchcock [11], and later by Koopmans [15]. Since then, numerous studies have been conducted on this subject. Researchers such as Mahapatra et al. [17], Midya and Roy [22], [21], and Maity and Roy [19] have investigated various facets of this problem and proposed diverse models for its optimization.

In addition to the aforementioned studies, several recent contributions have enriched the literature on fuzzy and multi-objective transportation problems. Roy et al. [26] investigated a multi-objective two-stage grey transportation problem using utility functions. Rivaz et al. [24] applied fuzzy goal programming to multi-objective transportation problems. Ebrahimnejad [9] proposed a simplified approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. Allahdadi and Rivaz [1] presented new results on rough interval linear programming with applications to scheduling and fixed-charge transportation problems. Finally, Ebrahimnejad and Verdegay [10] developed an efficient computational method for transportation problems based on type-2 intuitionistic fuzzy numbers. These studies highlight different approaches to handling uncertainty in transportation problems and complement the present work.

Fuzzy set theory has been presented as an effective tool for dealing with uncertainty in optimization problems. Since the parameters of the transportation problem, such as costs, supply, and demand, may be associated with uncertainty, the classical transportation problem is extended into a fuzzy transportation problem (FTP) using fuzzy numbers. This approach can assist decision-makers in making better decisions in real-world scenarios where accurate information is not available [22], [21], [19], [25].

In the present study, the fuzzy objective function and the TP destination demands are represented as triangular fuzzy numbers. These expectations are presented in a multi-choice format, and the decision-maker must, in alliance with the set conditions, allocate goods in such a way that the maximum profit is achieved and the destinations' demand can be fulfilled. Our proposed model involves fuzzy allocation, where decision variables are represented as fuzzy numbers. Furthermore, allocation at each node may not be mandatory and depends on the best fitness of the problem. In cases where allocation to a specific cell is not necessary, a deterministic zero priority goal is set. This model is solved using a multi-choice fuzzy goal programming approach, which can provide new insights into solving fuzzy transportation problems.

In real-world applications, parameters such as costs, supplies, and demands are often uncertain, making fuzzy set theory an effective tool to capture this imprecision. Goal programming (GP) provides a robust framework for solving multi-objective decision-making problems where competing goals must be balanced. However, classical GP models become challenging when the objectives and constraints themselves are fuzzy and when multiple goal levels must be considered simultaneously. To address these challenges, this paper proposes a multi-choice fuzzy goal programming model combined with two complementary solution techniques:

(i) a fuzzy least-squares linear regression method.

(ii) a fuzzy binary polynomial approximation for multi-choice goals.

This integration allows decision-makers to compare fuzzy and crisp parameters using the necessity measure (degree 0.8), ensuring both mathematical rigor and practical feasibility.

The concept of goal programming was first introduced by Charnes et al. [7] in the 1970s and has since become one of the primary approaches in decision-making analysis. This approach enables decision-makers to maximize the achievement of various goals in accordance with their priorities and requirements. Extensive research has been conducted in the field of goal programming, with authors such as Li et al. [16] and many others contributing to the development of various models and techniques in this field. These studies encompass investigations of different resource allocation methods, sensitivity analysis, and performance evaluation in multi-objective settings [5, 12, 23, 27].

Additionally, in light of the uncertainties that may arise in objective setting, studies have also been conducted in the field of multi-choice goal programming. Researchers such as Chang [6] and Maity [20] have investigated the modeling of fuzzy objectives and their integration into goal programming methods. These approaches help decision-makers make better decisions under conditions of uncertainty.

In the end, goal programming, as a robust analytical tool, can be applied across various fields—including resource management, production planning, transportation, and other sophisticated decision-making areas. This method not only assists decision-makers in moving closer to their objectives but also facilitates the evaluation and comparison of different alternatives.

The present study is an attempt to formulate a fuzzy goal programming (FGP) problem in which the decision variables are defined as multi-choice fuzzy numbers. In this model, the objective function is designed to incorporate multiple fuzzy goals, thereby allowing the decision-maker to consider various goals that may be ambiguous or uncertain. In this approach, the formulated model is solved using a multi-choice fuzzy goal programming method. One of the main advantages of utilizing multi-choice fuzzy numbers is that they provide more flexibility in both setting objectives and allocating resources.

The proposed model not only aids in selecting the optimal goal related to the objective function but also allows for optimal allocation of resources to various cells or units. This process involves identifying and prioritizing different goals, determining desirable values, and then finding the optimal resource allocation. Overall, by presenting a theoretical and practical framework to address sophisticated decision-making problems under uncertainty or ambiguity, the present study can assist decision-makers make better decisions across various fields, including resource management, production, and planning. By integrating goal programming theories with fuzzy number theory, this approach offers a robust tool for analyzing and solving multi-objective problems in real-world scenarios.

In real-world scenarios, particularly in optimization, uncertainty is a common phenomenon. One of the significant classes of optimization problems influenced by uncertainty is the transportation problem (TP). Owing to its unique characteristics, TP has garnered considerable attention across various fields, including supply chain management and urban planning. Investigation and analysis of ambiguous conditions and uncertainties in real-world transportation problems are among the most important and practical topics for researchers such as Ebrahimnejad [8] and Kaur and Kumar [14], [13]. Their studies demonstrate that key parameters such as costs, supply, and demand are often influenced by variable and imprecise factors. Consequently, many researchers have employed these parameters as imprecise variables or interval values.

However, in these studies [8], [14], [13], the concept of uncertainty is theoretically incorporated into the proposed models. These approaches are not primarily viewed as a practical approach for transportation problems and often fail to effectively address the real-world challenges.

Maity and Kumar [18] introduced and addressed fuzzy transportation problems using an alternative approach. Although their method employed multi-objective programming and a goal programming framework to address TP, it was found to be of low efficiency in solving sophisticated transportation problems. The key contributions and novel aspects of this research can be summarized as follows:

• Integration of fuzzy least-squares regression with multi-choice goal programming

We develop a unified framework that combines the fuzzy least-squares linear regression technique with a Fuzzy multi-choice goal programming (FMCGP) model. This integration enables simultaneous handling of multiple fuzzy goals while maintaining mathematical rigor in the optimization process.

• Fuzzy binary polynomial approximation for multi-choice goals

- We introduce a fuzzy binary polynomial approach to approximate multi-choice fuzzy parameters on the right-hand side of constraints. This method enhances the accuracy and flexibility of goal representation compared to classical fuzzy GP approaches.

• Necessity-based comparison for fuzzy-crisp inequalities

- All comparisons between fuzzy numbers and crisp values are carried out using the necessity measure with a necessity degree of 0.8, ensuring valid and consistent inequality handling throughout the model.

• Transportation example

- We apply the proposed methodology to a practical transportation investment problem, demonstrating improved feasibility and solution quality compared to existing methods in the literature.

Collectively, these innovations provide a significant step forward in solving fuzzy transportation problems and distinguish the present study from our previous works.

The overall structure of this paper is organized as follows. In Section 1, the introductory concepts are presented. Section 2 gives an account of the fuzzy transportation problem and analyzes this problem under real-world conditions. In Section 3, a mathematical model for the fuzzy transportation problem is introduced using a multi-choice fuzzy goal programming approach. Section 4 is dedicated to solving the proposed model by employing fuzzy binary polynomials and the fuzzy least squares linear regression. Finally, Section 5 solves a numerical example of a transportation problem and compares the results with those obtained from approach [18], and finally analyzes the results.

2. Fuzzy Transportation

The FTP model, with its novel approach, can be recognized as an effective tool for addressing sophisticated transportation problems under uncertainty. By focusing on fuzzy decision variables and optimizing customer demands, this model can assist decision-makers in achieving better outcomes in supply chain management. This novel approach may serve as a foundation for further research on fuzzy transportation and its applications in real-world scenarios.

This model is distinguished not only by its innovative approach in defining fuzzy decision variables but also by its specific focus on customer needs and the expected quantities of goods, thereby offering enhanced capabilities compared to classical models. This innovation has the potential to significantly impact decision-making in transportation and supply chain management, paving the path for future research.

The transportation planning problem is generally formulated as follows [18]:

$$
\begin{array}{lll}
\min & z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij} \leq a_i & i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} \geq b_j & j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0 & i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n.
\end{array} \tag{1}
$$

where x_{ij} denotes the decision variable representing the good transported from source i to destination j and c_{ij} is the transportation cost from source i to destination j per unit of goods. Additionally, a_i and b_j represent the supply quantity at source i and the demand at destination j respectively. It is important to note that the feasibility condition for the transportation problem is that:

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j.$$

That means the total supply across all sources must be greater than or equal to the total demand across all destinations.

In practical real-world applications, there may be situations where the expectations in the allocation cells of the transportation problem are considered as fuzzy goals. In such cases, the fuzzy transportation problem is formulated as follows [18]:

$$
(2)\qquad \begin{array}{lll}
\min & \widetilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\widetilde{x}_{ij}, & \\
\text{s.t.} & \sum_{j=1}^{n} \widetilde{x}_{ij} \lesssim a_i & i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} \widetilde{x}_{ij} \gtrsim b_j & j = 1, 2, \ldots, n, \\
& \widetilde{x}_{ij} \geq 0, & i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,
\end{array}
$$

where the variables c_{ij}, a_i and b_j are defined as in model (1), and $\widetilde{x}_{ij}$ are considered as triangular fuzzy numbers in the following form $\widetilde{x}_{ij} = (l_{ij}, m_{ij}, r_{ij})$, $i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n$.

2.1. **Fuzzy Transportation Using a Goal Programming Approach.** Multi-Choice Goal Programming (MCGP) is a robust approach in decision-making literature that enables decision-makers (DMs) to set multiple goal levels for each objective. In other words, in MCGP, each objective may have several goal levels, which can be defined either quantitatively or qualitatively. This approach allows decision-makers to strike a balance among multiple objectives instead of solely optimizing a single one, thereby making better decisions.

Multi-choice goal programming problem was first introduced by Chang [6]. As mentioned before, in such problems, the decision-maker is allowed to set several goal levels for each objective. In real-world scenarios, where precise information about the problem might be unavailable, multiple fuzzy choices are employed for each objective.

The fuzzy multi-choice goal programming (FMCGP) problem is formulated as follows:

$$
(3)\qquad \begin{array}{ll}
\min & \sum_{k=1}^{K} w_k(\eta_k + \rho_k), \\
\text{s.t.} & \widetilde{z}^k(x) + \rho_k - \eta_k = \widetilde{g}_k^r, \\
& \widetilde{g}_k^r \in \left\{ \widetilde{g}_k^1, \widetilde{g}_k^2, \ldots, \widetilde{g}_k^{R_j} \right\},\ k = 1, 2, \ldots, K, r = 1, 2, \ldots, R_k, \\
& x \in F,
\end{array}
$$

where $\widetilde{z}^k(x)$, $k = 1, 2, \ldots, K$ denotes the $k-$th objective function, and F is the feasible region. In order for the objective function in model (3) to be minimized, the distance of each objective function $\widetilde{z}^k(x)$ from the fuzzy goal $\widetilde{g}_k^r$, $r = 1, 2, \ldots, R_k$, $k = 1, 2, \ldots, K$ corresponding to this objective function should be minimized.

The mathematical model presented in this section provides a comprehensive framework for solving the goal-based transportation problem under fuzzy decision variables. By employing FMCGP, decision-makers can strike a good balance among various objectives and achieve sustainable economic, social and environmental outcomes. The fuzzy transportation problem with multiple fuzzy goals for each objective is formulated as follows:

$$(4)\quad \begin{array}{ll} \min & \sum_{k=1}^{K} w_k(\eta_k+\rho_k), \\ \text{s.t.} & \widetilde{z}^k(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{x}_{ij}+\rho_k-\eta_k=\widetilde{g}_k^r, \\ & \widetilde{g}_k^r\in\left\{\widetilde{g}_k^1,\widetilde{g}_k^2,\ldots,\widetilde{g}_k^{R_j}\right\},\ k=1,2,\ldots,K, r=1,2,\ldots,R_k, \\ & \sum_{j=1}^{n}\widetilde{x}_{ij}\lesssim a_i,\ i=1,2,\ldots,m, \\ & \sum_{i=1}^{m}\widetilde{x}_{ij}\gtrsim b_j,\ j=1,2,\ldots,n, \\ & \widetilde{x}_{ij}\geq 0,\ i=1,2,\ldots,m,\ j=1,2,\ldots,n. \end{array}$$

To solve the model (4), we first reformulate the MCGP model to a multi-choice goal programming model. In this process, auxiliary variables η_k and ρ_k are employed, and the model (4) is reformulated as follows:

$$(5)\quad \begin{array}{lll} \min & \sum_{k=1}^{K}\omega_k(\rho_k+\eta_k), & \\ \text{s.t.} & \widetilde{z}_k(x)+\eta_k-\rho_k=\widetilde{g}_k^r, & \\ & \widetilde{g}_k^r\in\left\{\widetilde{g}_k^1,\widetilde{g}_k^2,\ldots,\widetilde{g}_k^{R_K}\right\}, & k=1,2,\ldots,K,\ r=1,2,\ldots,R_k, \\ & \sum_{j=1}^{n}\widetilde{x}_{ij}\lesssim a_i & i=1,2,\ldots,m, \\ & \sum_{i=1}^{m}\widetilde{x}_{ij}\gtrsim b_j & j=1,2,\ldots,n, \\ & \widetilde{x}_{ij}\geq 0, & i=1,2,\ldots,m,\ j=1,2,\ldots,n. \end{array}$$

The weights are specified by the decision-makers, and it is assumed that they satisfy the normalization condition $\sum_{k=1}^{k} w_k=1$. The auxiliary variables η_k, ρ_k representing the positive and negative deviations from the k-th objective function, must be minimized. ω_k is the weight corresponding to the $k-$th objective function, for which varying positive values are determined according to the decision-maker's priority for the respective objective function.

To solve model (5), the constraints that have multi-choice fuzzy parameters on the right-hand side must first be transformed. For this purpose, the right-hand side of these constraints is approximated by a fuzzy function, thereby converting the problem into a typical fuzzy transportation problem. To approximate the right-hand side of the constraints, the fuzzy least squares method and a polynomial approach with fuzzy binary variables are employed, each of which is introduced in Section 3.

3. Solving the Proposed Model with Fuzzy Function Approximation

In the present study, to solve model (5), we first construct the ordered pairs $\left\{\left(1,\widetilde{g}_k^1\right),\left(2,\widetilde{g}_k^2\right),\ldots,\left(R_k,\widetilde{g}_k^{R_k}\right)\right\}$ based on the fuzzy values on the right-hand side of the fuzzy constraints. In practice, the decision maker specifies an ordinal ranking of the multi-choice goal levels according to preference or priority. These ordered levels are mapped to integer indices $r=1,2,\ldots,R_k$ to facilitate the construction of a binary polynomial or least-squares approximation. This approximation captures the trend of goal preferences and enables a continuous representation that can be optimized within the fuzzy goal programming framework. Different orderings may yield different approximations; this flexibility reflects the subjective nature of multi-choice decision making. Then, using the fuzzy binary approach and the fuzzy least squares method, we approximate the given polynomial points $\widetilde{p}_k(z)$ for $k = 1, 2, \ldots, K$ which satisfies the following condition:

$$\widetilde{p}_k(r) = \widetilde{g}_k^r, \qquad k = 1, 2, \ldots, K, \ r = 1, 2, \ldots, R_k. \tag{6}$$

3.1. **Fuzzy Binary Polynomial Approach.** Chang first introduced a binary polynomial based on the number of multi-choice parameters. A similar polynomial is required for the fuzzy scenario. The only difference between the fuzzy and deterministic approaches is that in the fuzzy method, the coefficients of the polynomial are fuzzy numbers, which results in the polynomial being fuzzy. In this approach, the set of multi-choice fuzzy parameters is replaced by a continuous fuzzy binary function [2–4].

In [3, 4], this polynomial is expressed for various numbers of choices. Since In the present study-and particularly in the numerical example-the fuzzy parameters on the right-hand side of the constraints are specified with two, three, or four fuzzy choices, this polynomial is introduced for these choices. Readers interested in a greater number of choices may consult [3, 4].

Case 1: Rk=2. In this case, we consider the ordered pairs $\left\{\left(1, \widetilde{g}_k^1\right), \left(2, \widetilde{g}_k^2\right)\right\}$ and the function $\widetilde{p}_k(z)$ is formulated as follows:

$$\widetilde{p}_k(z) = \widetilde{q}_k^1(1 - z) + \widetilde{g}_k^2 z, \tag{7}$$

where z is a binary variable taking the values 0 and 1.

Case 2: Rk=3. In this case, to construct the binary polynomial $\widetilde{p}_k(z)$ the ordered pairs $\left\{\left(1, \widetilde{g}_k^1\right), \left(2, \widetilde{g}_k^2\right), \left(3, \widetilde{g}_k^3\right)\right\}$ and the binary variables z_1, z_2 are required. In this case, the polynomial $\widetilde{p}_k(z)$ is defined in two different ways.

$$\widetilde{p}_k(z) = (1 - z_1)(1 - z_2)\widetilde{g}_k^1 + (1 - z_1)z_2\widetilde{g}_k^2 + z_1(1 - z_2)\widetilde{g}_k^3, \tag{8}$$

Where the binary variables z_1, z_2 satisfy the condition $z_1 + z_2 \leq 1$. Alternatively, the polynomial $\widetilde{p}_k(z)$ can be defined for binary variables with the condition $z_1 + z_2 \geq 1$ as follows:

$$\widetilde{p}_k(z) = (1 - z_1)z_2\widetilde{g}_k^1 + (1 - z_2)z_1\widetilde{g}_k^2 + z_1 z_2\widetilde{g}_k^3. \tag{9}$$

Case 3: Rk=4. In this case, the ordered pairs $\left\{\left(1, \widetilde{g}_k^1\right), \left(2, \widetilde{g}_k^2\right), \left(3, \widetilde{g}_k^3\right), \left(4, \widetilde{g}_k^4\right)\right\}$ are required to construct the binary polynomial $\widetilde{p}_k(z)$. Since there are four ordered pairs, binary variables z_1, z_2 are employed, giving the following formulation

$$\widetilde{p}_k(z) = z_1 z_2\widetilde{g}_k^1 + (1 - z_1)z_2\widetilde{g}_k^2 + z_1(1 - z_2)\widetilde{g}_k^3 + (1 - z_1)(1 - z_2)\widetilde{g}_k^4. \tag{10}$$

Now, model (5) can be reformulated as follows:

$$
\begin{array}{lll}
\min & \sum_{k=1}^{K} \omega_k(\rho_k+\eta_k), & \\
\text{s.t.} & \widetilde{z}_k(x)+\eta_k-\rho_k=\widetilde{p}_k(z), & \\
 & \sum_{j=1}^{n} \widetilde{x}_{ij} \lesssim a_i & i=1,2,\ldots,m, \\
 & \sum_{i=1}^{m} \widetilde{x}_{ij} \gtrsim b_j & j=1,2,\ldots,n, \\
 & \sum_{k=1}^{K} \omega_k = 1, & \\
 & z=0,1, & \\
 & \widetilde{x}_{ij} \geq 0, & i=1,2,\ldots,m,\ j=1,2,\ldots,n.
\end{array} \tag{11}
$$

3.2. **Fuzzy Least-Squares Linear Regression.** The least squares linear regression is one of the most fundamental techniques in numerical analysis and statistics, used to find the best linear approximation for a given set of data. This method seeks to minimize the sum of the squared deviations between the observed values and those predicted by the function. Here, we intend to introduce this approach to handle multiple fuzzy choices; to that end, Table 1 is considered for values $k=1,2,\ldots,K$.

Table 1. Data Points for the Fuzzy Least Squares Linear Regression Method.

r	1	2	3	...	R_k
$\widetilde{g}_k^r$	$\widetilde{g}_k^1$	$\widetilde{g}_k^2$	$\widetilde{g}_k^3$	...	$\widetilde{g}_k^{R_k}$

The objective is to find a first-degree polynomial (line) $\widetilde{p}_k(z)$ based on the data in Table 1 such that the distance between the line and the data points is minimized. To achieve this, assuming that $\widetilde{p}_k(z)=\widetilde{a}_0k+\widetilde{a}_1kz$, we define the error E as follows:

$$
E=\sum_{r=1}^{R_k}\left(\widetilde{g}_k^r-\widetilde{a}_{0k}-a_{1k}r\right)^2, \qquad k=1,2,\ldots,K. \tag{12}
$$

The necessary and sufficient condition for minimizing E is that its fuzzy derivative with respect to the fuzzy variables $\widetilde{a}_{1k}$ and $\widetilde{a}_{0k}$ equals zero. In such a case, for $k=1,2,K$, we have:

$$
\begin{cases}
\dfrac{\partial E}{\partial \widetilde{a}_{0k}}=0 \Rightarrow \widetilde{a}_{0k}\left(\sum_{r=1}^{R_k} r\right)+\widetilde{a}_{1k}\left(\sum_{r=1}^{R_k} r^2\right)=\sum_{r=1}^{R_k} r\widetilde{g}_k^r, \\[2ex]
\dfrac{\partial E}{\partial \widetilde{a}_{1k}}=0 \Rightarrow \widetilde{a}_{0k}R_k+\widetilde{a}_{1k}\left(\sum_{r=1}^{R_k} r\right)=\sum_{r=1}^{R_k} \widetilde{g}_k^r,
\end{cases} \tag{13}
$$

In Equation (13), the derivative is taken in the sense of fuzzy differentiation, which follows the extension principle for fuzzy-valued functions. Specifically, we consider the fuzzy error function E as a mapping from crisp decision variables to fuzzy numbers and use the generalized Hukuhara derivative. Setting $\dfrac{\partial E}{\partial \widetilde{a}_{0k}}=0$

and $\dfrac{\partial E}{\partial \widetilde{a}_{1k}} = 0$ provides the necessary and sufficient conditions to minimize the fuzzy squared deviation. This approach guarantees that the fuzzy regression coefficients $\widetilde{a}_{0k}$ and $\widetilde{a}_{1k}$ yield the least-squares.

By solving this system of equations for the fuzzy variables $\widetilde{a}_{1k}$ and $\widetilde{a}_{0k}$ the fuzzy polynomial $\widetilde{p}_k(z)$, which minimizes the squared distance from the data, is obtained. The coefficients $\widetilde{a}_{0k}$ and $\widetilde{a}_{1k}$ may take positive or negative fuzzy values depending on the underlying data. A positive fuzzy slope $\widetilde{a}_{1k}$ indicates a direct relationship between the index r and the fuzzy goal levels $\widetilde{g}_k^r$. In contrast, a negative fuzzy slope implies an inverse relationship. Interpreting these signs allows decision-makers to understand whether the goal levels increase or decrease over time. After constructing the polynomial $\widetilde{p}_k(z)$, the model (5) is reformulated as follows:

$$
\begin{array}{lll}
\min & \sum_{k=1}^{K} \omega_k(\rho_k + \eta_k), & \\
\text{s.t.} & \widetilde{z}_k(x) + \eta_k - \rho_k = \widetilde{p}_k(z), & \\
& \sum_{j=1}^{n} \widetilde{x}_{ij} \lesssim a_i & i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} \widetilde{x}_{ij} \gtrsim b_j & j = 1, 2, \ldots, n, \\
& \sum_{k=1}^{K} \omega_k = 1, & \\
& z = 1, 2, \ldots, R_k, & \\
& \widetilde{x}_{ij} \geq 0, & i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n.
\end{array}
\tag{14}
$$

Model (14) is then solved using LINGO software with a minimum degree of necessity of 0.8. Then, it applies a branch-and-bound procedure combined with linear programming techniques to obtain the optimal allocation of resources. This procedure ensures that all fuzzy decision variables satisfy the required feasibility levels while minimizing the weighted deviation in the multi-choice goals. The reported solution in Table 4 reflects the best compromise between the fuzzy profit objectives and the transportation constraints.

In the following, to evaluate the proposed methods, the example from [18] will be solved using these approaches and then the resulting outcomes will be compared and analyzed. Additionally, the advantages and disadvantages of the methods introduced in this paper and the method presented in [18] will be addressed.

4. Numerical Example

Example 4.1. *In this study, all comparisons between fuzzy and crisp quantities are performed using the necessity measure with degree* 0.8*, ensuring mathematically consistent and practically interpretable inequalities. The benchmark example adapted from [18] is employed only as an illustration of feasibility, and not as a direct superiority test, since the underlying problem formulations differ. Instead, the comparative analysis in Section 4 focuses on evaluating the proposed methods in terms of their feasibility, computational effort, and interpretability. In particular, the fuzzy binary polynomial approach and the fuzzy least-squares regression method are contrasted: the former provides a discrete* *choice representation of multi-choice goals, while the latter offers a continuous approximation. This dual perspective highlights the flexibility of the proposed framework and its ability to accommodate different decision-making requirements under uncertainty.*

Three investors, namely F_1, F_2, and F_3 intend to invest in two sites, A and B. The investment amounts for these investors are specified in a fuzzy manner, and appropriate planning must be conducted based on these amounts as well as the required constraints. The investment unit for each investor is as follows: F_1 has 1000 units, F_2 has 1100 units, and F_3 has 1200 units. The investment sites also have constraints; specifically, site A requires a minimum investment of 1600 units, while site B requires a minimum investment of 1650 units. Therefore, for each investor, it is essential to determine the optimal distribution of their available capital between the two sites, ensuring that both the minimum investment thresholds at each site are met and the investor's overall capital constraints are observed [18].

The constraints are as follows: an investor cannot invest more than their total available capital across sites A and B. Table 2 presents the investment amounts at the destinations (which are multi-choice fuzzy parameters).

Table 2. Required Amounts (in Units) and Deviations at the Sites.

	A	B
F_1	$(675, 700, 725)$ $(800, 850, 900)$ $(980, 1050, 1120)$ $(575, 600, 625)$	$(940, 980, 1020)$
F_2	$(750, 800, 850)$ $(610, 650, 690)$	$(475, 500, 525)$ $(370, 400, 430)$ $(570, 600, 630)$
F_3	$(790, 850, 910)$ $(650, 700, 750)$	$(525, 550, 575)$ $(420, 450, 480)$ $(950, 1050, 1150)$

In this problem, the positive and negative deviations are assumed to be identical. If no allocation is made at a particular node, an explicit zero allocation might occur; meaning that at each node there is an explicit zero choice that is not shown in Table 2, as this table only lists the nonzero values. The expected profit from the investment policy per 100 units of the destination is provided in Table 3.

Investors expect their profit to be no more than 170 units and no less than 150 units. Thus, it can be argued that the allocations at the sites are multi-choice fuzzy numbers. Based on the discussion above and the information provided in

Table 3. Profit Derived from Investment (per 100 units)

	A	B
F_1	4.5	4.0
F_2	5.0	6.0
F_3	5.5	5.0

Tables 2 and 3, the above problem is formulated as follows:

$$
\begin{array}{lll}
 & \min & \widetilde{z} = 4.5\widetilde{x}_{11} + 2\widetilde{x}_{12} + 5\widetilde{x}_{21} + 6\widetilde{x}_{2} + 5.5\widetilde{x}_{31} + 5\widetilde{x}_{32}, \\
 & \text{s.t.} & \widetilde{x}_{11} + \widetilde{x}_{12} \lesssim 1000, \\
 & & \widetilde{x}_{31} + \widetilde{x}_{22} \lesssim 1200, \\
(15) & & \widetilde{x}_{31} + \widetilde{x}_{32} \lesssim 1100, \\
 & & \widetilde{x}_{11} + \widetilde{x}_{21} + \widetilde{x}_{32} \gtrsim 1600, \\
 & & \widetilde{x}_{12} + \widetilde{x}_{22} + \widetilde{x}_{32} \gtrsim 1650, \\
 & & \widetilde{x}_{ij} \geq 0, \qquad i = 1,2,3, \ j = 1,2.
\end{array}
$$

Using the proposed method, model (15) *is then reformulated as follows:*

$$
\begin{array}{lll}
 & \min & 0.16n_{11} + 0.16n_{12} + 0.17n_{21} + 0.17n_{22} + 0.17n_{31} + 0.17n_{32}, \\
 & \text{s.t.} & y_{11} + n_{11} - p_{11} = \{(675,700,725),(800,850,900),(980,1050, \\
 & & \qquad 1120),(575,600,625)\}, \\
 & & y_{12} + n_{12} - p_{12} = (940,980,1020), \\
 & & y_{21} + n_{21} - p_{21} = \{(750,800,850),(610,650,690)\}, \\
 & & y_{22} + n_{22} - p_{22} = \{(475,500,525),(370,400,430),(570,600, \\
 & & \qquad 630)\}, \\
 & & y_{31} + m_{31} - p_{31} = \{(790,850,910),(650,700,750)\}, \\
(16) & & y_{32} + n_{32} + p_{32} = \{(525,550,575),(420,450,480),(950,1050, \\
 & & \qquad 1150)\} \\
 & & 4.5y_{11} + 4y_{12} + 5y_{21} + 6y_{22} + 5.5y_{31} + 5y_{32} \geq 150, \\
 & & 4.5y_{11} + 5y_{12} + 5y_{21} + 6y_{22} + 5.5y_{31} + 5y_{32} \leq 170, \\
 & & y_{11} + y_{12} \leq 1000, \\
 & & y_{21} + y_{22} \leq 1200, \\
 & & y_{31} + y_{32} \leq 1100, \\
 & & y_{11} + y_{21} + y_{31} \geq 1600, \\
 & & y_{12} + y_{22} + y_{32} \geq 1650.
\end{array}
$$

To solve model (16) *using the fuzzy binary polynomial approach, the multi-choice fuzzy parameters on the right-hand side of the constraints are approximated with a fuzzy binary polynomial that satisfies* (11)*. In doing so, model*

(16) *is reformulated as follows:*

$$\begin{array}{ll}
\min & 0.16n_{11}+0.16n_{12}+0.17n_{21}+0.17n_{22}+0.17n_{31}+0.17n_{32},\\
\text{s.t.} & y_{11}+n_{11}-p_{11}=(-535z_1z_2+225z_2+405z_1+575,-600z_1z_2+\\
& \qquad 250z_2+450z_1+600,-670z_1z_2+275z_2+495z_1+625),\\
& y_{12}+n_{12}-p_{12}=(940,980,1020),\\
& y_{21}+n_{21}-p_{21}=(610+140z_3,650+150z_3,690+160z_3),\\
& y_{22}+n_{22}-p_{22}=(-465z_4z_5+370z_4+570z_5,-500z_4z_5+400z_4\\
& \qquad +600z_5,-535z_4z_5+430z_4+630z_5),\\
& y_{31}+n_{31}-p_{31}=(650+140z_6,700+150z_6,750+160z_6),\\
& y_{32}+n_{32}-p_{32}=(-845z_7z_8+420z_7+950z_8,-950z_7z_8+450z_7\\
& \qquad +1050z_8,-1055z_7z_8+480z_7+1150z_8),\\
& 4.5y_{11}+4y_{12}+5y_{21}+6y_{22}+5.5y_{31}+5y_{32}\geq 150,\\
& 4.5y_{11}+4y_{12}+5y_{21}+6y_{22}+5.5y_{31}+5y_{32}\leq 170,\\
& y_{11}+y_{12}\leq 1000,\\
& y_{21}+y_{22}\leq 1200,\\
& y_{31}+y_{32}\leq 1100,\\
& z_4+z_5\leq 1,\\
& z_7+z_8\leq 1,\\
& y_{11}+y_{21}+y_{31}\geq 1600,\\
& y_{12}+y_{22}+y_{32}\geq 1650,\\
& z_i=0,1,\ i=1,2,\ldots,8.
\end{array}$$

The model presented above is then solved using LINGO software with a minimum necessity degree of 0.8, and the resulting outputs are presented in Table (4).

To solve model (16) *using the fuzzy least squares linear approach, we approximate the multi-choice fuzzy parameters on the right-hand side of the constraints with a fuzzy linear polynomial whose coefficients are determined by solving system* (14). *In doing so, model* (16) *is reformulated as follows:*

$$\begin{array}{ll}
\min & 0.16n_{11}+0.16n_{12}+0.17n_{21}+0.17n_{22}+0.17n_{31}+0.17n_{32},\\
\text{s.t.} & y_{11}+n_{11}-p_{11}=(787.5-12z_{11},825-10z_{11},862.5-8z_{11}),\\
& y_{12}+n_{12}-n_{12}=(940,980,1020),\\
& y_{21}+n_{21}-p_{21}=(890-140z_{21},950-150z_{21},1010-160z_{21}),\\
& y_{22}+n_{22}-p_{22}=(343.3+47.5z_{22},400+50z_{22},423.3+52.5z_{22}),\\
& y_{31}+n_{31}-p_{31}=(930-140z_{31},1000-150z_{31},1070-160z_{31}),\\
& y_{32}+n_{32}-p_{32}=(206.6+212.5z_{32},183.3+250z_{32},160+\\
& \qquad 287.5z_{32}),4.5y_{11}+4y_{12}+5y_{21}+6y_{22}+5.5y_{31}+5y_{32}\geq 150,\\
& 4.5y_{11}+4y_{12}+5y_{21}+6y_{22}+5.5y_{31}+5y_{32}\leq 170,\\
& y_{11}+y_{12}\leq 1000,\\
& y_{21}+y_{22}\leq 1200,\\
& y_{31}+y_{32}\leq 1100,\\
& y_{11}+y_{21}+y_{31}\geq 1600,\\
& y_{12}+y_{22}+y_{32}\geq 1650,\\
& z_{11}=0,1,2,3,\ z_{21}=0,1,\ z_{22}=0,1,2,\ z_{31}=0,1,\ z_{32}=0,1,2.
\end{array}$$

The model presented above is then solved using LINGO software with a minimum necessity degree of 0.8, and the results are presented in Table 4. The results obtained from the proposed method in this paper, as well as those from the method in [18], are shown in Table 4.

TABLE 4. Model's Solution Using the Our Proposed Method and the Method of [19].

	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}
[18] Article	1000	0	610	590	0	1100
Binary method	84	916	776	424	740	360
Least squares method	228	772	682	518	740	360

Example 4.2. *In this example, four investors intend to allocate their capital across three investment sites. The investment amounts and corresponding limits for each investor are represented as fuzzy triangular numbers, reflecting the inherent uncertainty in both supply and demand. Similarly, the required amounts at each site and the profit values per 100 investment units are provided as fuzzy data in Tables 5 and 6. As in the previous case, the decision-maker must determine an optimal allocation that satisfies all fuzzy constraints while maximizing the overall return. However, compared with the first example, the present model involves four investors and three destinations, resulting in a higher-dimensional decision space and a more complex fuzzy structure.*

TABLE 5. Required Amounts (in Units) and Deviations at the Sites.

	A	B	C
F_1	$(600, 700, 800)$ $(500, 600, 700)$ $(800, 900, 1000)$	$(950, 1000, 1050)$ $(700, 800, 900)$	$(500, 600, 700)$ $(700, 800, 1000)$
F_2	$(600, 650, 700)$ $(900, 950, 1000)$ $(550, 600, 650)$	$(300, 400, 500)$ $(450, 500, 550)$ $(850, 900, 950)$ $(750, 850, 950)$	$(650, 700, 750)$ $(950, 1000, 1050)$
F_3	$(800, 850, 900)$ $(650, 700, 750)$ $(750, 800, 850)$	$(900, 1000, 1100)$	$(1000, 1100, 1200)$ $(800, 900, 1000)$
F_4	$(600, 650, 700)$ $(750, 800, 850)$	$(850, 900, 950)$ $(950, 1050, 1150)$ $(700, 800, 900)$	$(500, 600, 700)$ $(700, 800, 900)$ $(900, 1000, 1100)$

In this study, it is assumed that the positive and negative deviations are equal in magnitude. When no allocation is assigned to a specific node, it is interpreted as a zero allocation. This zero value is not explicitly displayed in the tables, since only the nonzero fuzzy allocation options are reported.

Table 6. Profit Derived from Investment (per 100 units).

	A	B	C
F_1	5	6	7
F_2	5.5	4.5	6.5
F_3	4	5	6
F_4	7	6	6.5

Based on the information provided in Tables 5 and 6, the above problem is formulated as follows:

$$\begin{aligned}
&\widetilde{x}_{21}+\widetilde{x}_{22}+\widetilde{x}_{23}\leq 1800,\\
&\widetilde{x}_{31}+\widetilde{x}_{32}+\widetilde{x}_{33}\leq 2100,\\
&\widetilde{x}_{41}+\widetilde{x}_{42}+\widetilde{x}_{43}\leq 1500,\\
&\widetilde{x}_{11}+\widetilde{x}_{21}+\widetilde{x}_{31}+\widetilde{x}_{41}\geq 2500,\\
&\widetilde{x}_{12}+\widetilde{x}_{22}+\widetilde{x}_{32}+\widetilde{x}_{42}\geq 2200,\\
&\widetilde{x}_{13}+\widetilde{x}_{23}+\widetilde{x}_{33}+\widetilde{x}_{43}\geq 2100,\\
&\widetilde{x}_{ij}\geq 0,\ \ i=1,2,3,4,\ \ j=1,2,3.
\end{aligned}\tag{17}$$

Using the proposed method, model (17) *is then reformulated as follows:*

$$\begin{aligned}
\min\quad & 0.1n_{11}+0.05n_{12}+0.1n_{13}+0.05n_{21}+0.1n_{22}+0.05n_{23}+\\
&\qquad 0.1n_{31}+0.05n_{32}+0.1n_{33}+0.1n_{41}+0.1n_{42}+0.1n_{43},\\
\text{s.t.}\quad & y_{11}+n_{11}-p_{11}=\{(600,700,800),(500,600,700),(800,900,1000)\},\\
& y_{12}+n_{12}-p_{12}=\{(950,1000,1050),(700,800,900)\},\\
& y_{13}+n_{13}-p_{13}=\{(500,600,700),(700,800,1000)\},\\
& y_{21}+n_{21}-p_{21}=\{(600,650,700),(900,950,1000),(550,600,650)\},\\
& y_{22}+n_{22}-p_{22}=\{(300,400,500),(450,500,550),(850,900,950),\\
&\qquad (750,850,900)\},\\
& y_{23}+n_{23}-p_{23}=\{(650,700,750),(950,1000,1050)\},\\
& y_{31}+n_{31}-p_{31}=\{(800,850,900),(650,700,750),(750,800,850)\},\\
& y_{32}+n_{32}-p_{32}=(900,1000,1100),\\
& y_{33}+n_{33}-p_{33}=\{(1000,1100,1200),(800,900,1000)\},\\
& y_{41}+n_{41}-p_{41}=\{(600,650,700),(750,800,850)\},\\
& y_{42}+n_{42}-p_{42}=\{(850,900,950),(950,1050,1150),(700,800,900)\},\\
& y_{43}+n_{43}-p_{43}=\{(500,600,700),(700,800,900),(900,1000,1100)\},\\
& y_{11}+y_{12}+y_{13}\leq 2000,\\
& y_{21}+y_{22}+y_{23}\leq 1800,\\
& y_{31}+y_{32}+y_{33}\leq 2100,\\
& y_{41}+y_{42}+y_{43}\leq 1500,\\
& y_{11}+y_{21}+y_{31}+y_{41}\leq 2500,\\
& y_{12}+y_{22}+y_{32}+y_{42}\geq 2200,\\
& y_{13}+y_{23}+y_{33}+y_{43}\leq 2100.
\end{aligned}$$

In this method, the multi-choice fuzzy parameters on the right-hand side of the constraints were approximated using the fuzzy binary polynomials. The coefficients of these polynomials are fuzzy numbers, while the binary variables represent the possible choices associated with each parameter. The reformulated model was then solved using the fuzzy goal programming framework, and the obtained results are presented in Table 7 (Binary method column).

$$
\begin{array}{ll}
\min & 0.1n_{11}+0.05n_{12}+0.1n_{13}+0.05n_{21}+0.1n_{22}+0.05n_{23}+0.1n_{31}+ \\
& \qquad 0.05n_{32}+0.1n_{33}+0.1n_{41}+0.1n_{42}+0.1n_{43}, \\
\text{s.t.} & y_{11}+n_11-p_{11}=(600+200z_1-100z_2-700z_1z_2, 700+200z_1-100z_2 \\
& \qquad -800z_1z_2, 800+200z_1-100z_2-900z_1z_2), \\
& y_{12}+n_{12}-p_{12}=(950-250z_3, 1000-200z_3, 1050-1050z_3), \\
& y_{13}+n_{13}-p_{13}=(500+200z_4, 600+200z_4, 700+300z_4), \\
& y_{21}+n_{21}-p_{21}=(550+350z_5+350z_6-900z_5z_6, 600+350z_5+350z_6 \\
& \qquad -950z_5z_6, 650+350z_5+350z_6-1000z_5z_6), \\
& y_{22}+n_{22}-p_{22}=(750+100z_7-300z_8-550z_7z_8, 850+50z_7-350z_8 \\
& \qquad -550z_7z_8, 950+0z_7-400z_8-550z_7z_8), \\
& y_{23}+n_23-p_23=(650+300z_9, 700+300z_9, 750+300z_9), \\
& y_{31}+n_31-p_31=(650+100z_{10}+100z_{11}-750z_{10}z_{11}, 700+100z_{10}+ \\
& \qquad 100z_{11}-800z_{10}z_{11}, 750+100z_{10}+100z_{11}-850z_{10}z_{11}), \\
& y_{32}+n_{32}-p_{32}=(900, 1000, 1100), \\
& y_{33}+n_{33}-p_{33}=(1000-200z_{12}, 110-200z_{12}, 1200-200z_{12}), \\
& y_{41}+n_{41}-p_{41}=(600+150z_{13}, 650+150z_{13}, 700+15z_{13}), \\
& y_{42}+n_{42}-p_{42}=(700+150z_{14}+250z_{15}-950z_{14}z_{15}, 800+150z_{14}+ \\
& \qquad 250z_{15}-1050z_{14}z_{15}, 900+150z_{14}+250z_{15}-1150z_{14}z_{15}), \\
& y_{43}+n_{43}-p_{43}=(500+400z_{16}+200z_{17}-900z_{16}z_{17}, 600+400z_{16}+ \\
& \qquad 200z_{17}-1000z_{16}z_{17}, 700+400z_{16}+200z_{17}-1100z_{16}z_{17}), \\
& z_i \in \{0,1\},\ i=1,2,\ldots,17, \\
& y_{11}+y_{12}+y_{13} \le 2000, \\
& y_{21}+y_{22}+y_{23} \le 1800, \\
& y_{31}+y_{32}+y_{33} \le 2100, \\
& y_{41}+y_{42}+y_{43} \le 1500, \\
& y_{11}+y_{21}+y_{31}+y_{41} \le 2500, \\
& y_{12}+y_{22}+y_{32}+y_{42} \le 2200, \\
& y_{13}+y_{23}+y_{33}+y_{43} \le 2100, \\
& z_1+z_2 \le 1, \\
& z_5+z_6 \le 1, \\
& z_{10}+z_{11} \le 1, \\
& z_{14}+z_{15} \le 1, \\
& z_{16}+z_{17} \le 1.
\end{array}
$$

In the second method, the fuzzy multi-choice parameters were approximated through a first-degree fuzzy polynomial derived by the least squares approach. The coefficients of the polynomial were determined determined to minimize minimize the total squared deviation between the observed fuzzy data and the approximated function. This transformation simplified the structure of the model while preserving its fuzzy nature. The corresponding results of this approach are also reported in Table 7 (Least squares method column). Both models were *solved with a minimum necessity degree of* 0.8.

$$\begin{array}{ll}
\min & 0.1n_{11}+0.05n_{12}+0.1n_{13}+0.05n_{21}+0.1n_{22}+0.05n_{23}+0.1n_{31}+\\
& 0.05n_{32}+0.1n_{33}+0.1n_{41}+0.1n_{42}+0.1n_{43},\\
\text{s.t.} & y_{11}+n_{11}-p_{11}=(433.3+100z_1,533.3+100z_1,633.33+100z_1),\\
& y_{12}+n_{12}-p_{12}=(950-250z_2,1000-200z_2,1050-150z_2),\\
& y_{13}+n_{13}-p_{13}=(500+200z_3,600+200z_3,700+300z_3),\\
& y_{21}+n_{21}-p_{21}=(466.67+150z_4,516.67+150z_4,566.67+150z_4),\\
& y_{22}+n_{22}-p_{22}=(125+200z_5,200+200z_5,275+200z_5),\\
& y_{23}+n_{23}-p_{23}=(650+300z_6,700+300z_6,750+300z_6),\\
& y_{31}+n_{31}-p_{31}=(566.67+100z_7,616.67+100z_7,666.67+100z_7),\\
& y_{32}+n_{32}-p_{32}=(900,1000,1100),\\
& y_{33}+n_{33}-p_{33}=(1000-200z_8,1100-200z_8,1200-200z_8),\\
& y_{41}+n_{41}-p_{41}=(600+150z_9,650+150z_9,700+150z_9),\\
& y_{42}+n_{42}-p_{42}=(566.67+125z_{10},666.67+125z_{10},766.67+125z_{10}),\\
& y_{43}+n_{43}-p_{43}=(300+200z_{11},400+200z_{11},500+200z_{11}),\\
& z_1,z_4,z_7,z_{11},z_{12}=0,1,2,\\
& z_2,z_3,z_6,z_9,z_10=0,1,\\
& z_5=0,1,2,3,\\
& y_{11}+y_{12}+y_{13}\leq 2000,\\
& y_{21}+y_{22}+y_{23}\leq 1800,\\
& y_{31}+y_{32}+y_{33}\leq 2100,\\
& y_{41}+y_{42}+y_{43}\leq 1500,\\
& y_{11}+y_{21}+y_{31}+y_{41}\geq 2500,\\
& y_{12}+y_{22}+y_{32}+y_{42}\geq 2200,\\
& y_{13}+y_{23}+y_{33}+y_{43}\geq 2100.
\end{array}$$

Table 7. The Solution of the Model Using the Proposed Methods.

	y_{11}	y_{12}	y_{13}	y_{31}	y_{32}	y_{33}	y_{21}	y_{22}	y_{23}	y_{41}	y_{42}	y_{43}
Binary method	580	840	580	690	530	880	590	490	690	640	420	800
Least squares method	640	780	580	613	545	690	606	185	780	640	646	213

The obtained results are compared and analyzed in the conclusion.

5. Conclusion

The present study developed an advanced fuzzy multi-choice goal programming model for solving transportation and resource allocation problems under uncertainty. By integrating fuzzy set theory with goal programming, the proposed framework successfully captured the inherent imprecision in real-world decision environments where supply, demand, and profit parameters cannot be expressed deterministically. Two fuzzy function approximation approaches based on the fuzzy binary polynomial and the fuzzy least-square method were employed to handle the multi-choice fuzzy parameters. The binary polynomial method offered a flexible representation of discrete fuzzy choices, while the least-squares linear approach provided a smoother and computationally simpler approximation of fuzzy data. Both techniques were implemented with a necessity degree of 0.8, ensuring solution reliability within an acceptable confidence range. The numerical investigations, including both the initial and the extended examples, demonstrated the consistency and adaptability of the proposed model. When the dimensionality of the problem increased by additional investors and destinations, the model preserved its feasibility and yielded interpretable results that satisfied all fuzzy constraints. The comparison of results between the two examples revealed that variations in the fuzzy structure of the data significantly influence the final allocation pattern, yet both approaches produced coherent and stable solutions. This confirms that the model effectively adapts to different system complexities and uncertainty levels. Overall, the findings validate that the proposed fuzzy multi-choice goal programming framework provides a reliable and generalizable tool for real-world transportation and investment planning problems. Its capability to balance multiple fuzzy objectives and maintain constraint satisfaction under uncertainty highlights its potential for practical decision-support applications in logistics, production, and financial planning. Future research may extend this framework by incorporating intelligent optimization algorithms or hybrid fuzzy techniques to enhance performance in large-scale or dynamic problems.

References

[1] Allahdadi, M., & Rivaz, S. (2024). Some new results on rough interval linear programming problems and their application to scheduling and fixed-charge transportation problems. RAIRO-Operations Research, 58(5), 3697-3714.

[2] Arami, Z., Arabameri, M., & Mishmast Nehi, H. (2023). Fuzzy approximating functions and its application in solving fuzzy multi-choice linear programming models. Journal of Mathematical Modeling, 11(4), 649-663. doi: 10.22124/JMM.2023.24269.2178

[3] Biswal, M. P., & Acharya, S. (2009). Multi-choice multi-objective linear programming problem. Journal of Interdisciplinary Mathematics, 12(5), 606-637. doi.org/10.1080/09720502.2009.10700650

[4] Biswal, M. P., & Acharya, S. (2009). Transformation of a multi-choice linear programming problem. Applied Mathematics and Computation, 210(1), 182-188. doi.org/10.1016/j.amc.2008.12.080

[5] Chang, C. T. (2007). Multi-choice goal programming. Omega, 35(4), 389-396. doi.org/10.1016/j.omega.2005.07.009

[6] Chang, C. T. (2008). Revised multi-choice goal programming. Applied Mathematical Modelling, 32(12), 2587-2595. doi.org/10.1016/j.apm.2007.09.008

[7] Charnes, A., Cooper, W. W., & Ferguson, R. O. (1955). Optimal estimation of executive compensation by linear programming. Management Science, 1(2), 138-151. doi.org/10.1287/mnsc.1.2.138.

[8] Ebrahimnejad, A. (2015). An improved approach for solving fuzzy transportation problem with triangular fuzzy numbers. Journal of Intelligent & Fuzzy Systems, 29(2), 963-974. doi: 10.3233/IFS-151625.

[9] Ebrahimnejad, A. (2014). A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. Applied Soft Computing, 19, 171-176.

[10] Ebrahimnejad, A., & Verdegay, J. L. (2016). An efficient computational approach for solving type-2 intuitionistic fuzzy numbers based transportation problems. International Journal of Computational Intelligence Systems, 9(6), 1154-1173.

[11] Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. Journal of Mathematics and Physics, 20(1-4), 224-230.doi.org/10.1002/sapm1941201224

[12] Ignizio, J. P. (1976). Goal programming and extensions. (Lexington Books: Lexington, MA, 1976).

[13] Kaur, A., & Kumar, A. (2012). A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. Applied Soft Computing, 12(3), 1201-1213. doi.org/10.1016/j.asoc.2011.10.014

[14] Kaur, A., & Kumar, A. (2011). A new method for solving fuzzy transportation problems using ranking function. Applied Mathematical Modelling, 35(12), 5652-5661. doi.org/10.1016/j.apm.2011.05.012

[15] Koopmans, T. C. (1949). Optimum utilization of the transportation system. Econometrica: Journal of the Econometric Society, 136-146. doi.org/10.2307/1907301

[16] Lee, A. H., Kang, H. Y., Yang, C. Y., & Lin, C. Y. (2010). An evaluation framework for product planning using FANP, QFD and multi-choice goal programming. International Journal of Production Research, 48(13), 3977-3997. doi.org/10.1080/00207540902950845

[17] Mahapatra, D. R., Roy, S. K., & Biswal, M. P. (2013). Multi-choice stochastic transportation problem involving extreme value distribution. Applied Mathematical Modelling, 37(4), 2230-2240. doi.org/10.1016/j.apm.2012.04.024

[18] Maity, G., & Roy, S. K. (2017). Solving fuzzy transportation problem using multi-choice goal programming. Discrete Mathematics, Algorithms and Applications, 9(06), 1750076. doi.org/10.1142/S1793830917500768

[19] Maity, G., & Roy, S. K. (2014). Solving multi-choice multi-objective transportation problem: a utility function approach. Journal of uncertainty Analysis and Applications, 2, 1-20. doi. www.juaa-journal.com/content/2/1/11

[20] Maity, G., & Roy, S. K. (2016). Solving multi-objective transportation problem with interval goal using utility function approach. International Journal of Operational Research, 27(4), 513-529. doi.org/10.1504/IJOR.2016.080143

[21] Midya, S., & Roy, S. K. (2017). Analysis of interval programming in different environments and its application to fixed-charge transportation problem. Discrete Mathematics, Algorithms and Applications, 9(03), 1750040. doi:10.1142/S1793830917500409

[22] Midya, S., & Roy, S. K. (2014). Solving single-sink, fixed-charge, multi-objective, multi-index stochastic transportation problem. American Journal of Mathematical and Management Sciences, 33(4), 300-314. doi.org/10.1080/01966324.2014.942474

[23] Narasimhan, R. (1980). Goal programming in a fuzzy environment. Decision Sciences, 11(2), 325-336. doi.org/10.1111/j.1540-5915.1980.tb01142.x

[24] Rivaz, S., Nasseri, S. H., & Ziaseraji, M. (2020). A fuzzy goal programming approach to multiobjective transportation problems. Fuzzy information and engineering, 12(2), 139-149.

[25] Roy, S. K., & Maity, G. (2017). Minimizing cost and time through single objective function in multi-choice interval valued transportation problem. Journal of Intelligent & Fuzzy Systems, 32(3), 1697-1709. doi:10.3233/JIFS-151656

[26] Roy, S. K., Maity, G., & Weber, G. W. (2017). Multi-objective two-stage grey transportation problem using utility function with goals. Central European Journal of Operations Research, 25(2), 417-439.

[27] Tamiz, M., Jones, D., & Romero, C. (1998). Goal programming for decision making: An overview of the current state-of-the-art. European Journal of Operational Research, 111(3), 569-581. doi.org/10.1016/S0377-2217(97)00317-2

Zahra Arami
Orcid number: 0009-00005252-4453
Department of Mathematics
University of Sistan and Baluchestan
Zahedan, Iran
Email address: zahraarami22@gmail.com

Maryam Arabameri
Orcid number: 0000-0003-0269-4547
Department of Mathematics
University of Sistan and Baluchestan
Zahedan, Iran
Email address: arabameri@math.usb.ac.ir

Hasan Mishmast Nehi
Orcid number: 0000-0002-7573-1751
Department of Mathematics
University of Sistan and Baluchestan
Zahedan, Iran
Email address: hmnehi@hamoon.usb.ac.ir